Quantum dark solitons in the one-dimensional Bose gas

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Dark and grey soliton-like states are shown to emerge from numerically constructed superpositions of translationally-invariant eigenstates of the interacting Bose gas in a toroidal trap. The exact quantum many-body dynamics reveals a density depression with superdiffusive spreading that is absent in the mean-field treatment of solitons (see Fig. 1). A simple theory based on finite-size bound states of holes with quantum-mechanical center-of-mass motion quantitatively explains the time-evolution of the superposition states and predicts quantum effects that could be observed in ultra-cold gas experiments. The soliton phase step is shown to be a key ingredient of an accurate finite size approximation, which enables us to compare the theory with numerical simulations. The fundamental soliton width, an invariant property of the quantum dark soliton, is shown to deviate from the Gross-Pitaevskii predictions in the interacting regime and vanishes in the Tonks-Girardeau limit.

In addition to the one-dimensional Bose gas in the Lieb-Liniger model [1], we also consider the one-dimensional Fermi gas with attractive delta-function interactions (Yang-Gaudin model) [2]. The corresponding Bethe-ansatz equations are solved for finite particle number and in the thermodynamic limit in order to obtain the *yrast* dispersion, i.e. the dispersion relation of the eigenstates of lowest energy for given momentum. Properties corresponding to the soliton-like nature of the yrast excitations are calculated including the missing particle number, phase step, and inertial and physical masses. The inertial to physical mass ratio, which is related to the frequency of oscillations in a trapped gas, is found to be unity in the limits of strong and weak attraction and fall to ≈ 0.78 in the crossover regime. This result is contrasted by one-dimensional mean field theory, which predicts a divergent mass ratio in the weakly attractive limit. By means of an exact mapping our results also predict the existence and properties of dark-soliton-like excitations in the super Tonks-Girardeau gas. The prospects for experimental observations are briefly discussed.

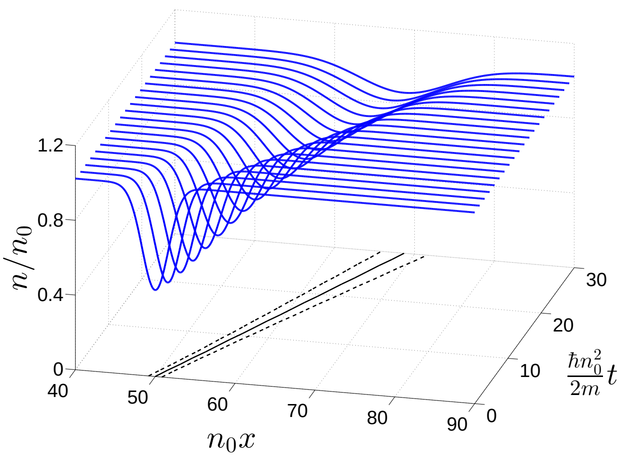

Figure 1: Time evolution of the quantum dark soliton constructed as a Gaussian superposition of yrast eigenstates of the Lieb-Liniger model with $N = 100$ particles at the intermediate interaction strength $\gamma = 1$. The solid line tracks the minimum of the dip and the dashed lines on either side of it are displaced by half of the soliton's width.

[1] Sophie S. Shamailov and Joachim Brand. Quantum dark solitons in the one-dimensional Bose gas. arXiv:1805.07856 (2018).

[2] Sophie S. Shamailov and Joachim Brand. Dark-soliton-like excitations in the Yang-Gaudin gas of attractively interacting fermions. *New J. Phys.*, 18(7):075004 (2016).