Trapping of bodies by the gravitational waves endowed with angular momentum

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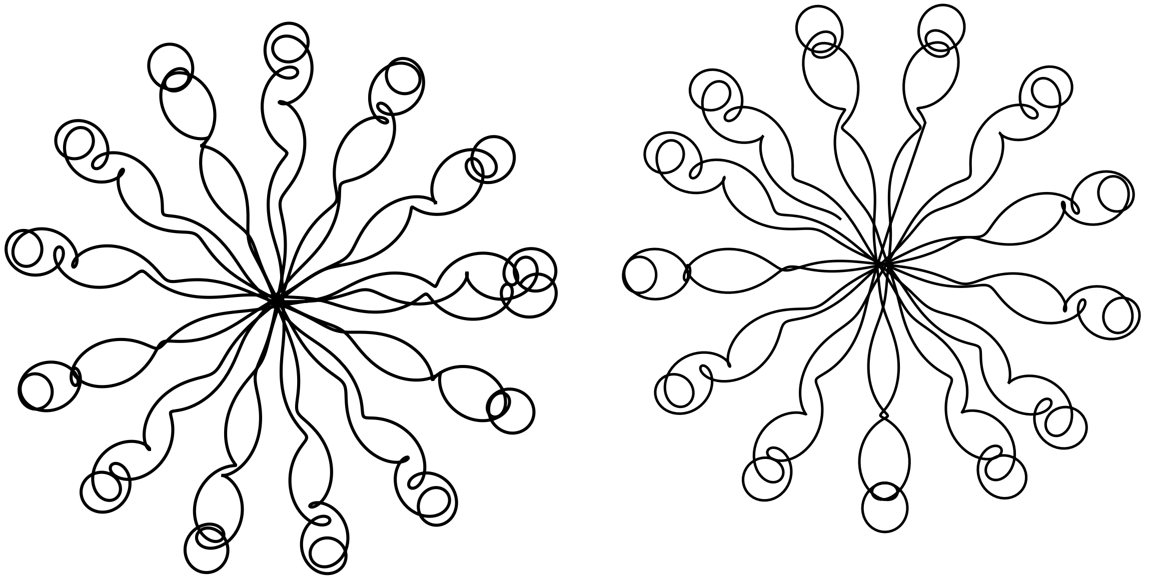
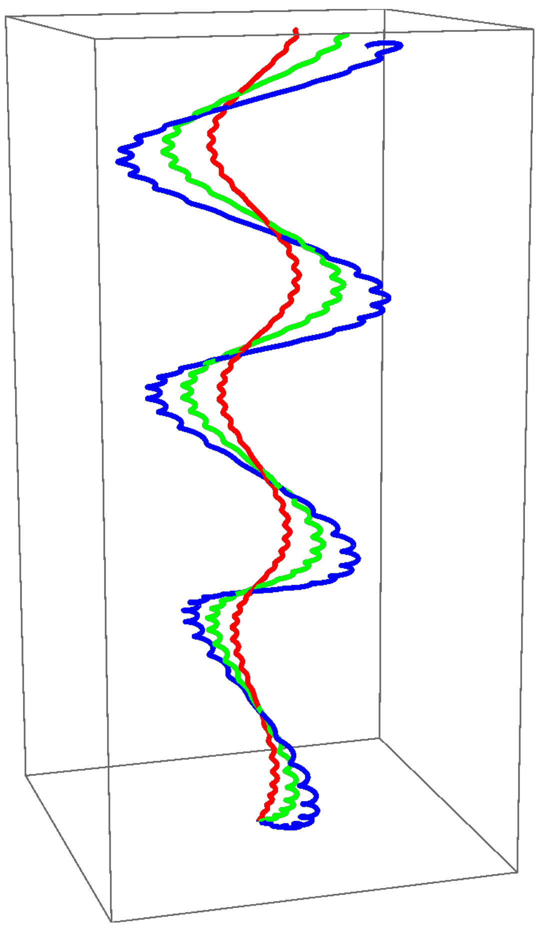
Prologue

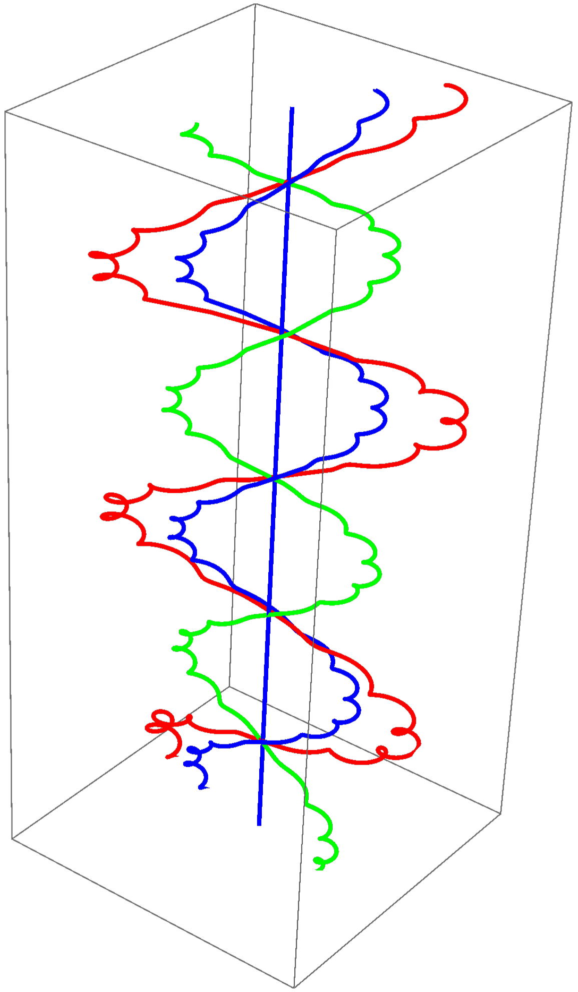
"What the principal investigator would like to be"trapped"?
If these are elementary particles,
then gravity is much too weak,
as compared to other forces at those scales
If they are massive bodies
(e.g. stars or compact objects),
then there are no physically motivated
sufficiently strong gravitational waves
to achieve this"

Electromagnetism **Gravitation**



Electromagnetism **Gravitation**





Angular momentum of a binary system of two equal masses

$$L = \sqrt{Gm^3 r} = \frac{Gm^2}{c}\sqrt{\frac{2r}{r_S}}$$

Example: Black holes with 30 solar masses spiraling at the distance of 10 Schwarzschild radii have the angular momentum hundred thousand times the angular momentum of the Earth on its orbit around the Sun

Angular momentum luminosity
for a binary system of two masses

$$\frac{dL}{dt} = \frac{32\sqrt{2}}{5} \frac{G^{7/2}}{c^5} \frac{m^{9/2}}{r^{7/2}} = \frac{4mc^2}{5(r/r_S)^{7/2}}$$

Example: Black holes with 30 solar masses spiraling at the distance of 10 Schwarzschild radii emit per second six million times the angular momentum of the Earth on its orbit around the Sun

Bessel beams carry angular momentum

Bessel beams in electromagnetism and gravity
can be obtained as derivatives of the superpotential

$$\boxed{\chi_{\lambda M q_z q_\perp}(\rho, \phi, z, t) = e^{-i\lambda(\omega_q t - q_z z - M\phi)} J_M(q_\perp \rho)}$$

Quantum numbers:

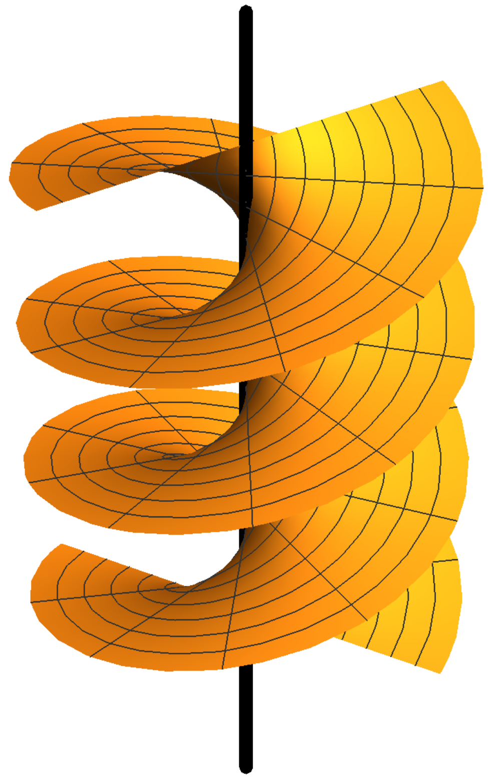
$\hbar\lambda$ - helicity $\quad \hbar\omega_q$ - energy

$\hbar M$ - z-component of total angular momentum

$\hbar q_z$ - z-component of momentum

$\hbar q_\perp$ - transverse momentum

Constant phase surface



Geodesic motion

$$m_{\text{inert}} \frac{d^2\xi^\mu}{d\tau^2} + m_{\text{grav}} \Gamma^\mu_{\alpha\beta} \frac{d\xi^\alpha}{d\tau} \frac{d\xi^\beta}{d\tau} = 0$$

τ is the proper time

Galileo-Einstein $\boldsymbol{m}_{\textbf{inert}} = \boldsymbol{m}_{\textbf{grav}}$

Mass disappears Motion is universal

$$\frac{d^2\xi^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{d\xi^\alpha}{d\tau} \frac{d\xi^\beta}{d\tau} = 0$$

The simplest geodesic trajectory

Experts in numerical gravity claim that
the dominant contribution to gravitational radiation
emitted by inspiraling binaries
comes from the $M = 2$ component of radiation
and I will consider only this case

Owing to the axial symmetry of Bessel beams
there is one natural candidate for a geodesic: the z-axis

Uniform motion along the z-axis

4-velocity has the form:

$$\frac{d\xi^\alpha}{d\tau} = \{u^0, 0, 0, u^3\}$$

Relevant components of the Christoffel symbol vanish

$$\Gamma^\mu_{\alpha\beta} = 0 \; (\alpha = 0, 3 \;\; \beta = 0, 3) \;\Rrightarrow\; \Gamma^\mu_{\alpha\beta} \frac{d\xi^\alpha}{d\tau} \frac{d\xi^\beta}{d\tau} = 0$$

Therefore, the motion is uniform

$$\boxed{\frac{d^2\xi^\mu}{d\tau^2} = 0}$$

Geodesic deviation

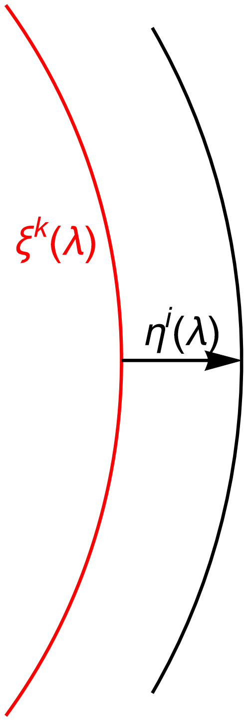
Is the geodesic trajectory along the z-axis stable?

The answer is obtained from
the equation of geodesic deviation

$$\boxed{\frac{d^2\eta^\mu}{d\tau^2} = R^\mu{}_{\alpha\beta\nu}\frac{d\xi^\alpha}{d\tau}\frac{d\xi^\beta}{d\tau}\eta^\nu}$$

$R^\mu{}_{\alpha\beta\nu}$ Riemann curvature tensor

What is geodesic deviation?



Reference geodesic is drawn in red

Riemann curvature tensor

$$R^1{}_{001} = -2k_+^4 B_{M-2} - 4k_\perp^2 k_+^2 B_M - 2k_\perp^4 B_{M+2},$$

$$R^1{}_{002} = 2k_+^4 B_{M-2} - 2k_\perp^4 B_{M+2},$$

$$R^1{}_{003} = -4k_\perp k_+^3 B_{M-1} - 4k_\perp^3 k_+ B_{M+1},$$

$$R^2{}_{002} = 2k_+^4 B_{M-2} - 4k_\perp^2 k_+^2 B_M - 2k_\perp^4 B_{M+2},$$

$$\dots$$

where $B_M = AJ_M(k_\perp\rho)\cos(k_\parallel + M\phi - \omega t)$

A is the amplitude that determines the strength

Looks very complicated, however...

Motion near the reference trajectory

Reference trajectory $\xi^i(\tau)$ along the z-axis

$$\frac{d^2\eta^1}{dt^2} = -\gamma\omega^2(\eta^1\cos(\omega t) + \eta^2\sin(\omega t)),$$

$$\frac{d^2\eta^2}{dt^2} = -\gamma\omega^2(\eta^1\sin(\omega t) - \eta^2\cos(\omega t)),$$

$$\frac{d^2\eta^3}{dt^2} = 0.$$

In the frame rotating with $\omega/2$ Coriolis force overcomes repulsion

$$\frac{d^2\eta^1}{dt^2} = (1/4-\gamma)\omega^2\eta^1 + \omega\frac{d\eta^2}{dt}$$
$$\frac{d^2\eta^2}{dt^2} = (1/4+\gamma)\omega^2\eta^2 - \omega\frac{d\eta^1}{dt}$$

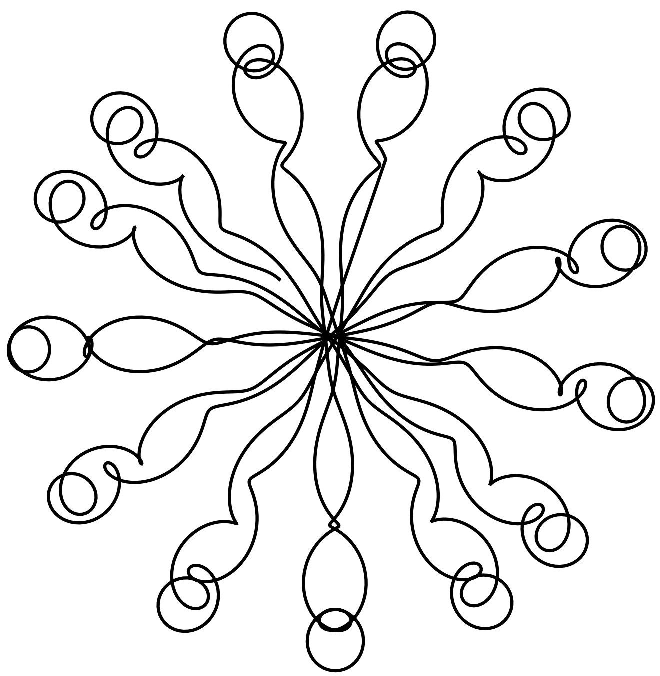
Eigenfrequencies in the rotating frame:

$$\Omega_\pm = \omega\sqrt{1/4 \pm \gamma}$$

In the inertial frame

The motion in the inertial frame
is characterized by 4 frequencies:

$$\Omega^{\pm}_{\pm} = \omega(1/2 \pm \sqrt{1/4 \pm \gamma})$$



Conclusion

Gravitational waves may carry with them
all kinds of cosmic debris
The trapping of these bodies near the beam axis
is due to the Coriolis force