New Zealand INSTITUTE for Advanced Study

CTCP Centre for Theoretical Chemistry and Physics

Quantum dark solitons in the one-dimensional Bose gas

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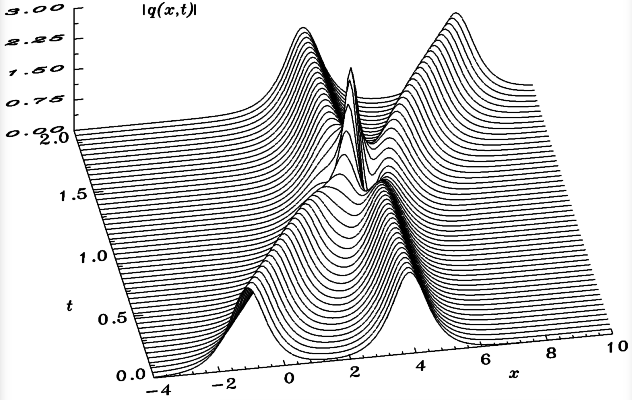
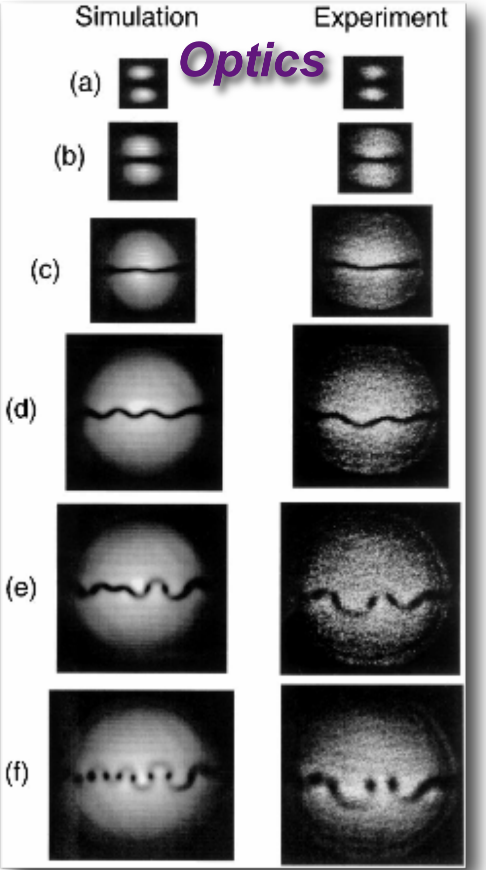
Humboldt Kolleg – Controlling quantum matter: from ultracold atoms to solids– July 2018

Sophie S. Shamailov

Solitons

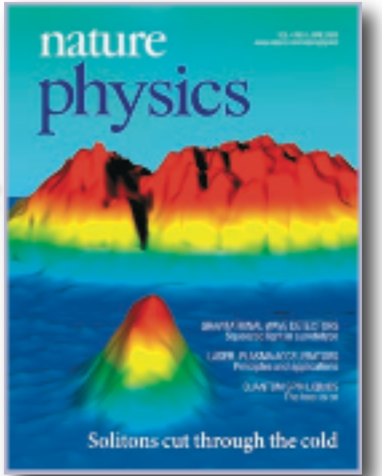




Tikhonenko et al. (1996)

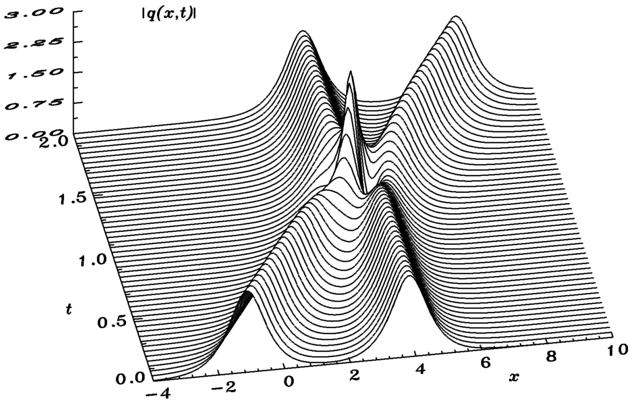
credit: Alex Kasman

BEC



Sengstock group (2008)

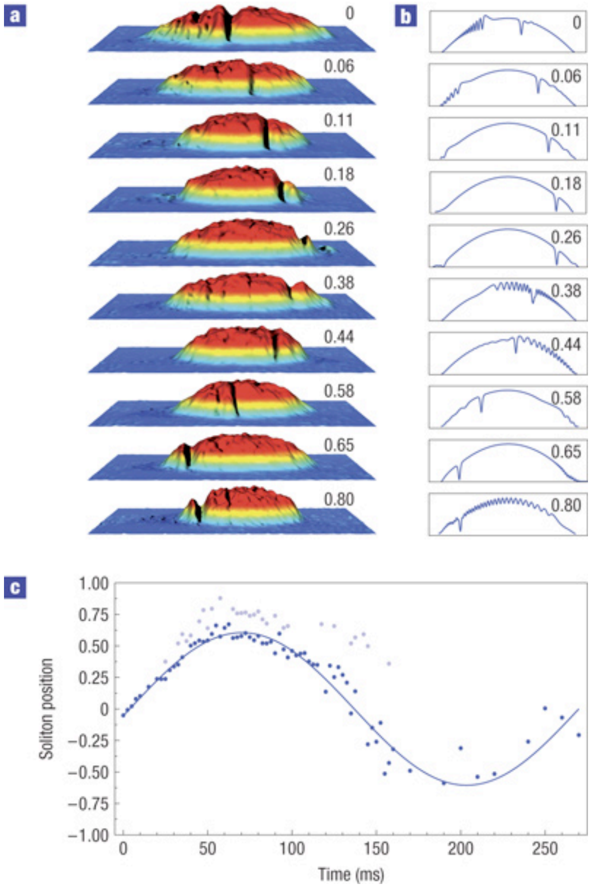
Solitons are solutions of nonlinear partial differential equations.



Can we find solitons in strongly-correlated quantum fluids?
What are their properties?

Let's find them in the one-dimensional Bose gas.

Dark soliton oscillations in BEC experiment



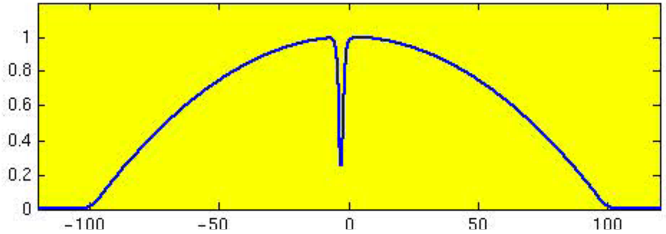
Hamburg Experiment: Becker et al. (2008)

Solitons in trapped BEC oscillate more slowly than COM

$$\frac{M_{\rm in}}{M_{\rm ph}} = \left(\frac{T_s}{T_{\rm trap}}\right)^2 = 2$$

Theory:
- Busch, Anglin PRL (2000)
- Konotop, Pitaevskii, PRL (2004)

Experiment:
- Becker et al. Nat. Phys. (2008)
- Weller et al. PRL (2008)



Movie credits: Nick Parker

Dark solitons

- Mean field (classical) theory:
  Defocussing nonlinear Schrödinger (Gross-Pitaevskii) equation

$$i\frac{\partial}{\partial t}u(x,t) = [-\frac{1}{2}\frac{\partial^2}{\partial x^2} + g|u|^2]u(x,t)$$

Number of depleted particles

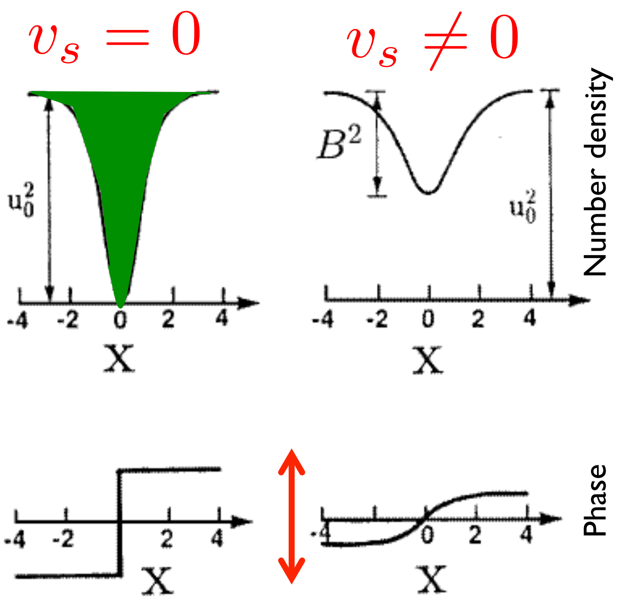
$$N_{\rm d} = \int [n(x) - n_{\rm bg}]dx$$

Dark and grey soliton solution (g>0):
Tsuzuki, JLTP (1971)

Superfluid phase step $\Delta\phi$

Effective mass, length scale

Velocity



Kivshar, Luther-Davis (1998)

The one-dimensional Bose gas

Lieb-Liniger model: Bosons with contact interactions in one dimension

$$H = -\frac{\hbar^2}{2m}\sum_{j=1}^{N}\frac{d^2}{dx_j^2} + g\sum_{\langle i,j\rangle}\delta\left(x_i - x_j\right)$$

Bethe ansatz wave function

$$\langle\{x_i\}|\{k_i\}\rangle = \sum_{\mathcal{P}\in S(N)} a(\mathcal{P})\exp\left(\sum_i k_{\mathcal{P}(i)}x_i\right)$$

$$k_j + \frac{1}{L}\sum_l 2\arctan\frac{k_j - k_l}{mgh^{-2}} = \frac{2\pi}{L}I_j$$

k_j - Rapidities/quasimomenta

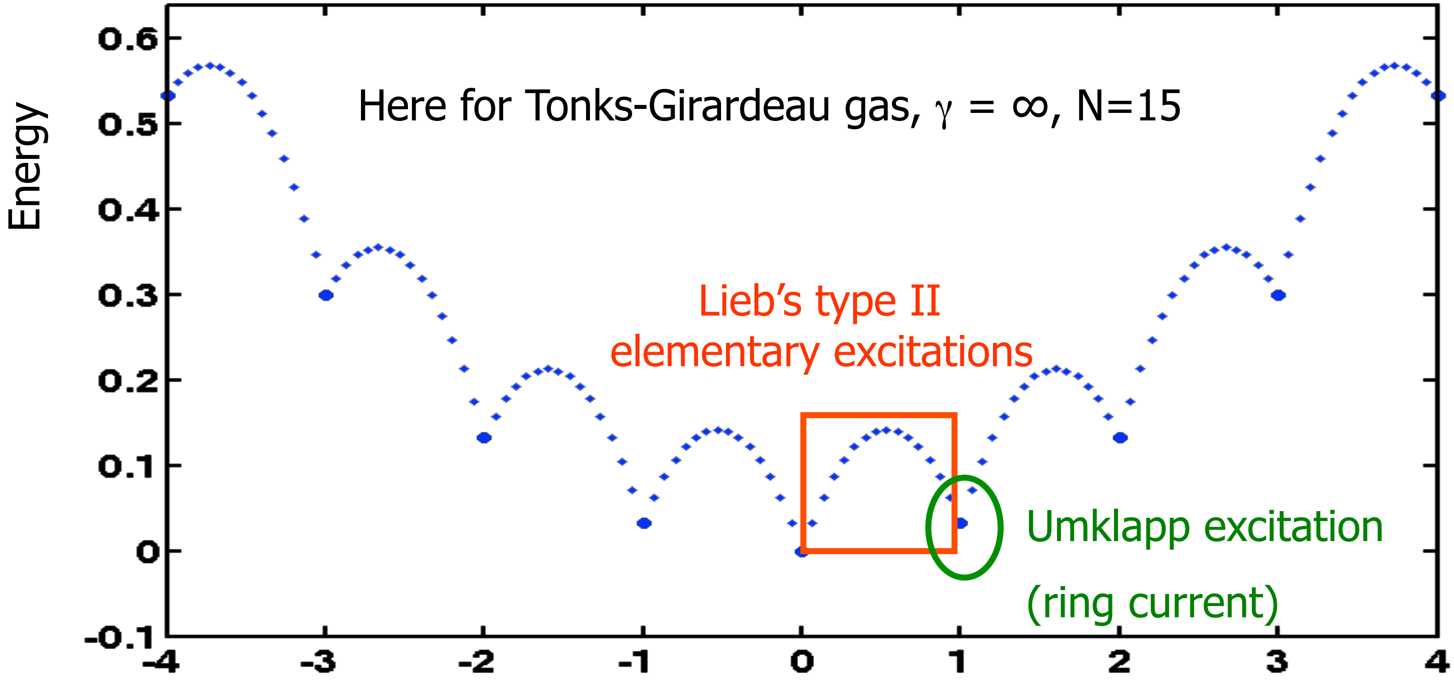
I_j - Integer quantum numbers

N - number of bosons

$$P_{\text{tot}} = \hbar\sum_{j=1}^{N}k_j,$$

$$E_{\text{tot}} = \frac{\hbar^2}{2m}\sum_{j=1}^{N}k_j^2.$$

Low-lying excitation spectrum (yrast states)



Momentum $\frac{P}{2\pi\hbar n}$

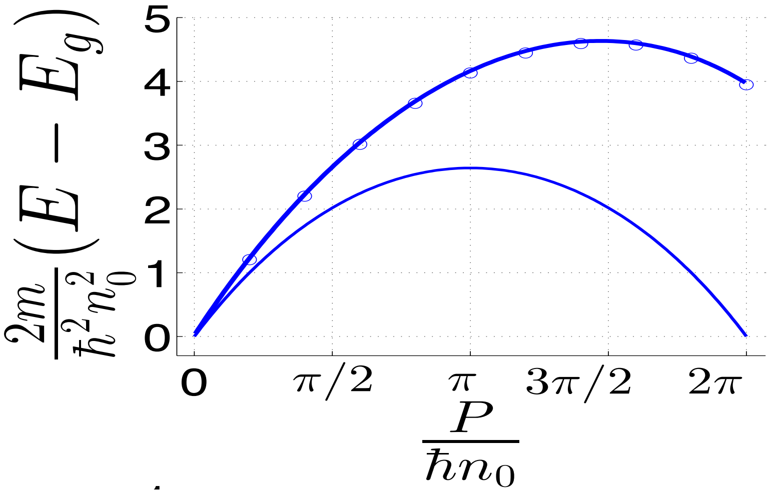
Experimental probe of dynamic structure factor Fabbri et al. (2015)

Eigenstates are translationally invariant!

Where are the solitons?

Comparison of dispersion relations

Takayama, Ishikawa, JPSP (1980):
Asymptotically (weak interaction, thermodynamic limit) is
GP dark soliton congruent with yrast dispersion of Lieb-Liniger model



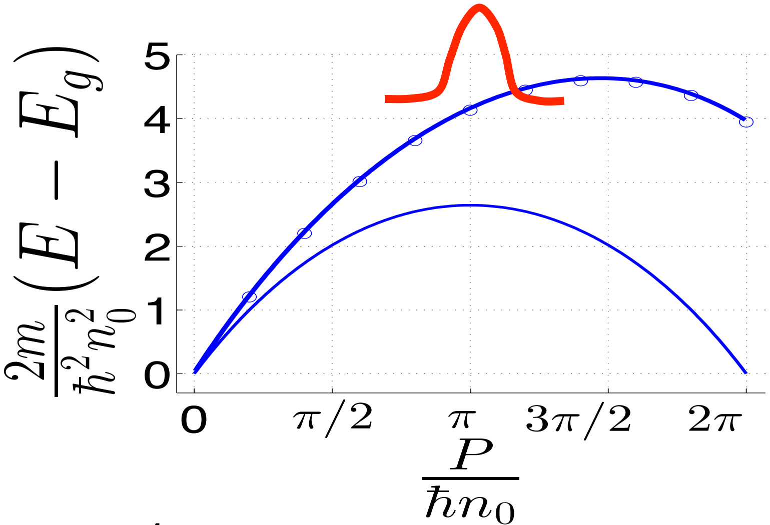
Circles: finite system (ring), N = 10, γ=1

Thermodynamic limit, N = ∞, γ=1

$$\gamma = \frac{gm}{n_0 \hbar^2}$$

How to get over the translational invariance of the eigenstates?

- Syrwid, Sacha, *PRA* (2015): Soliton emerges during particle measurement.
- Sato et al. *NJP* (2012, 2016): Localised density dip by superposition of *all* yrast eigenstates
- Our proposal: **Gaussian wave packet of yrast states**



Soliton velocity:

$$v_s = \frac{dE_s}{dp_c}$$

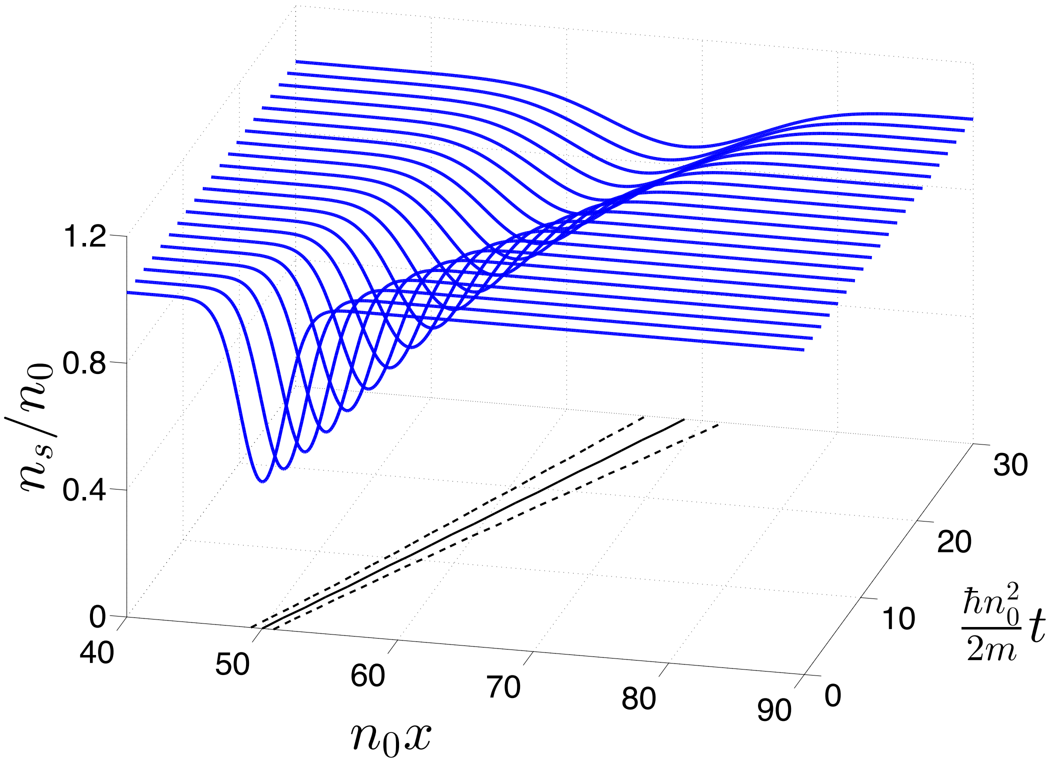
Fialko, Delattre, JB, Kolovsky (2012)
Shamailov, Brand, *arXiv:1805.07856*

Simulating time evolution

$$
\begin{aligned}
n(x,t) &= \langle P_0(t)|\hat{\rho}(x)|P_0(t)\rangle \\
&= \sum_{p,q} C_q^{P_0 *} C_p^{P_0} \underline{\langle q, \mathrm{yr}|\hat{\rho}(0)|p, \mathrm{yr}\rangle} \\
&\quad \times \exp[i(p-q)x/\hbar - i(E_p - E_q)t/\hbar],
\end{aligned}
$$

The form factor is calculated by determinantal formula from the rapidities. Formula derived from algebraic Bethe ansatz: Slavnov (1989), Korepin (1982), Caux (2007)

Time evolution of Gaussian wave packet (exact)



$N = 100$

$\gamma = 1$

Shamailov, Brand, *arXiv:1805.07856*

Time evolution of quantum dark soliton

Use the following ansatz, in analogy to quantum bright solitons:

$$\Delta x^2(t) = {\sigma_{\mathrm{fs}}}^2 + \sigma_{\mathrm{CoM}}^2(t),$$

$$\sigma_{\mathrm{CoM}}^2(t) = \sigma_0^2 \left[1 + \left(\frac{\hbar t}{2M\sigma_0^2}\right)^2\right]$$

where

$$\Delta x^2 = N_{\mathrm{d}}^{-1} \int (x - \langle x \rangle)^2 \left[n(x) - n_0\right] dx$$

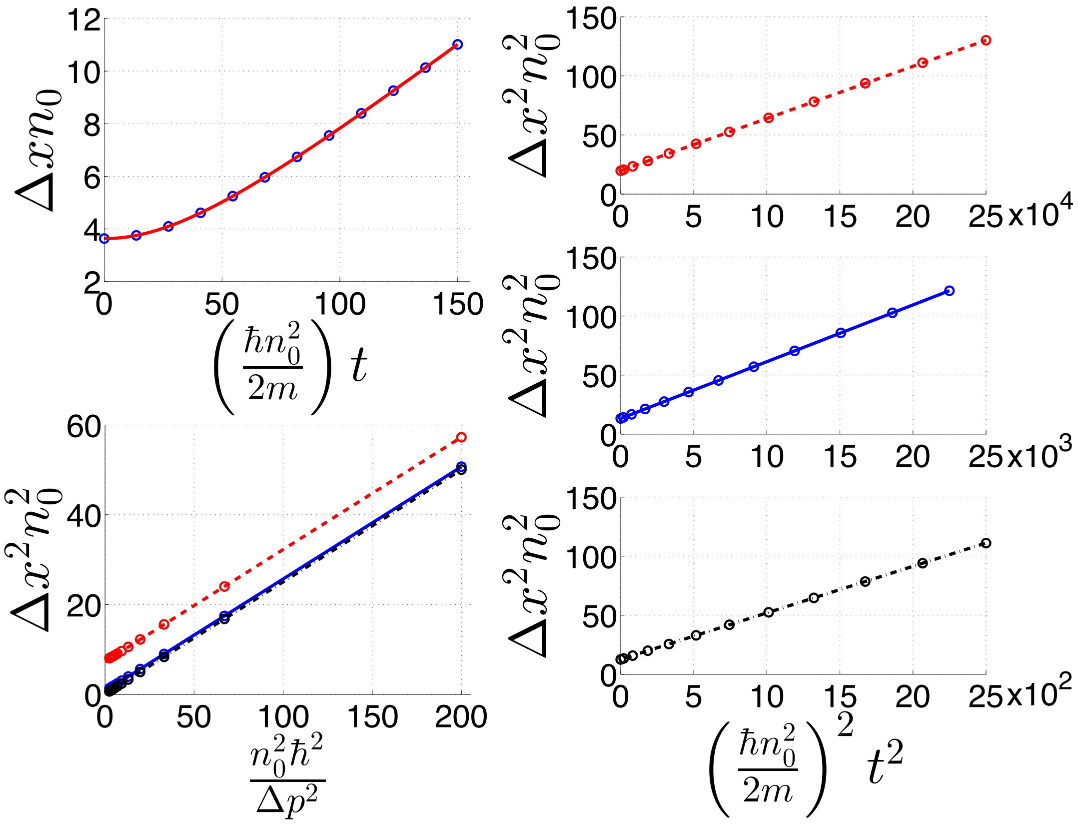
$$N_{\mathrm{d}} = \int \left[n(x) - n_0\right] dx$$

$$\sigma_0^2 = \frac{\hbar^2}{4\Delta P^2}$$

Ballistic spreading of the CoM – fit two parameters: $\sigma_{\mathrm{fs}}^2, M$

Shamailov, Brand, *arXiv:1805.07856*

Fits to the numerical time evolution



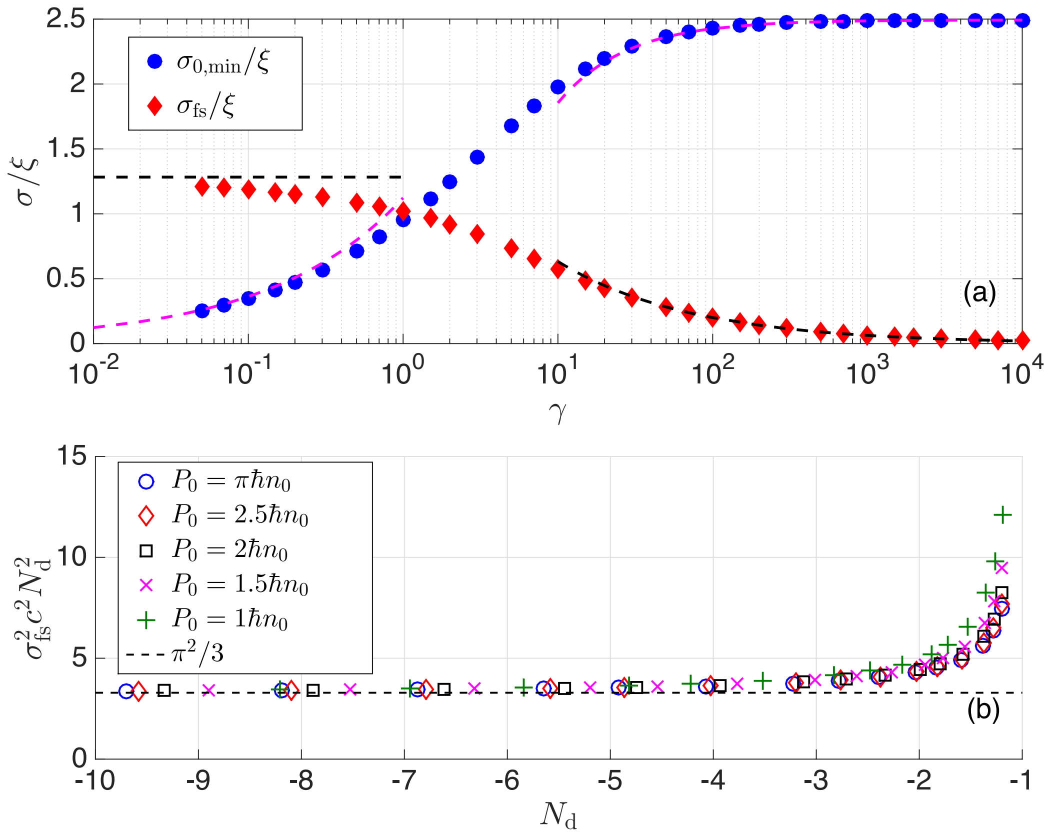
$\gamma = 0.1$

$\gamma = 1$

$\gamma = 10$

Shamailov, Brand, *arXiv:1805.07856*

The fundamental soliton width

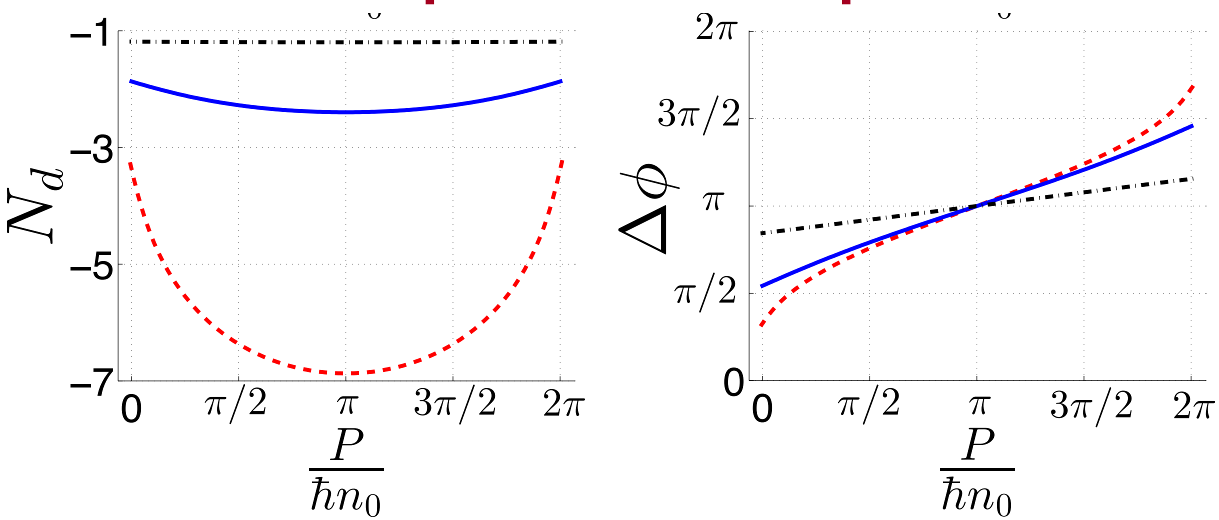


Soliton width from GP theory

$$\sigma_{\rm GP}^2 = \frac{\pi^2}{3\gamma^2 n_0^2 N_d^2}$$

Shamailov, Brand, *arXiv:1805.07856*

Particle number depletion and phase step



$$N_{\mathrm{d}} = -\left(1 - \frac{v_{\mathrm{s}}^2}{c^2}\right)^{-1}\left(\frac{\partial E_{\mathrm{s}}}{\partial \mu} + \frac{v_{\mathrm{s}} P}{mc^2}\right)$$

$$P = m v_s N_d + \frac{1}{2}\hbar n_0 \Delta\phi$$

So does the phase step mean anything here?

Shamailov, Brand, *arXiv:1805.07856*

Yes, the phase step is very important.

In ring geometry (periodic box), the phase step demands backflow current.

$$v_{\rm cf} = \frac{\hbar \Delta\phi}{mL}$$

Energy and momentum have corrections from Galilean boost.

$$E_{\rm s}^{N}(P) \approx E_{\rm s}^{\infty}(P) + P_s v_{\rm cf} + \frac{1}{2} N m v_{\rm cf}^2 + \frac{N_d^2}{2L}\frac{d\mu}{dn_0}$$

$$P = m v_s N_d + \frac{1}{2}\hbar n_0 \Delta\phi$$

$$P_s = m v_s N_d$$

Phase uncertainty is inherited from momentum uncertainty.

Comparison of dispersion relations

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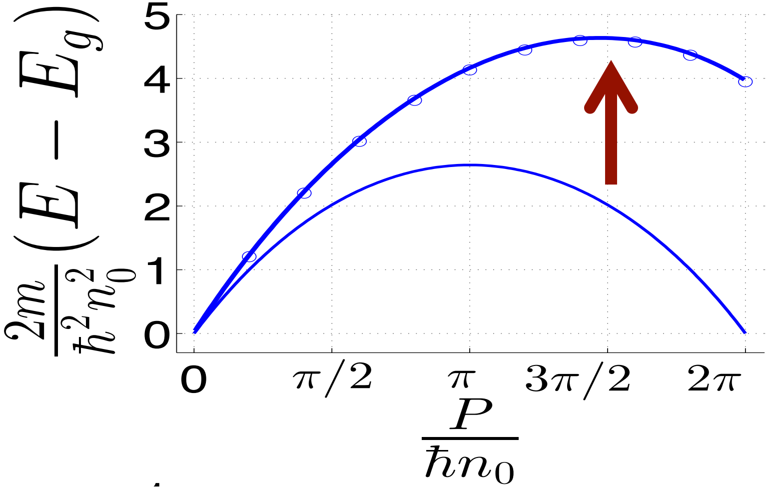


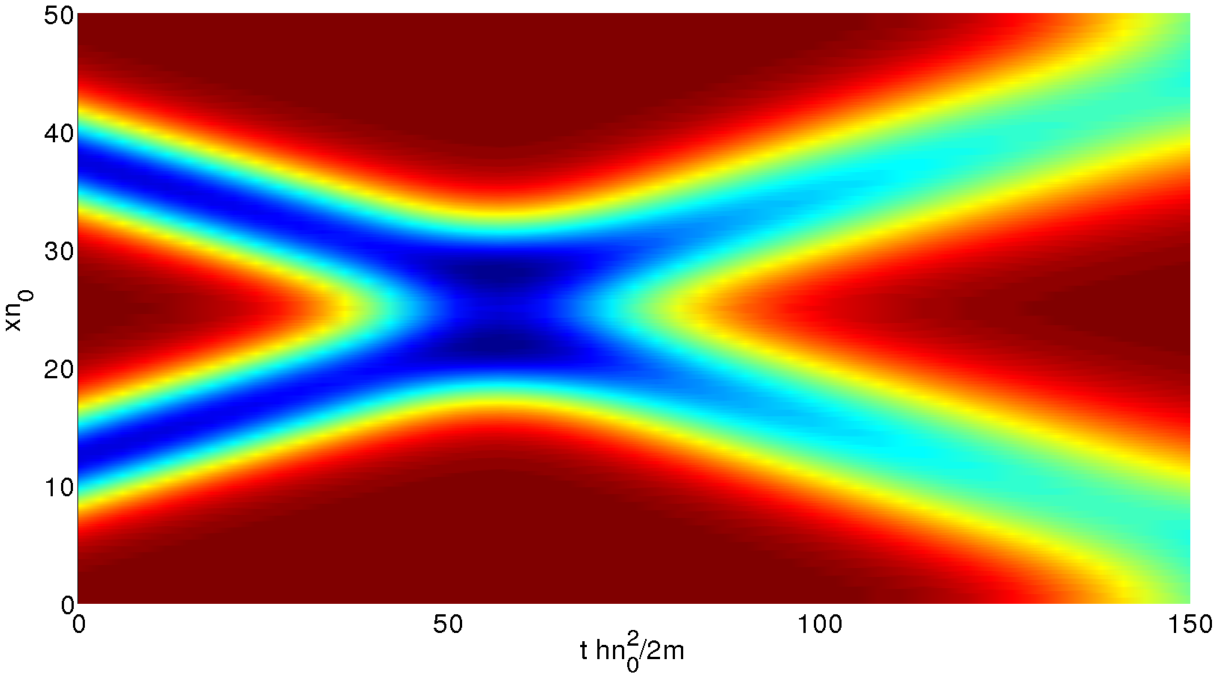
Circles: finite system (ring), N = 10, γ=1

Thermodynamic limit, $N = \infty$, γ=1

$$\gamma = \frac{gm}{n_0 \hbar^2}$$

Energy corrections from phase step provide an excellent approximation of finite size dispersions.

Quantum soliton collisions



What have we learned…

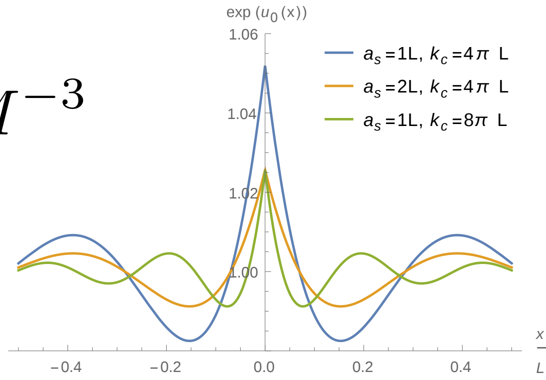
- Even in the absence of true long-range order, solitons persist
- Solitons behave like quantum-mechanical bound states of (a noninteger number of) holes
- The phase step is relevant for the backflow current – on a global scale

…beyond the 1D Bose gas?

- Yrast excitation spectrum may hold the key to soliton-like excitations even in non-integrable models: many properties can be obtained as derivatives
- Spreading of density is controlled by effective mass
- Generalisations to fermions, long-range interactions, …

Other projects / future:

- Stochastic exact diagonalisation in Fock space for ultra-cold atoms:
  Collaboration with Ali Alavi (Stuttgart) on FCIQMC
- Accelerating Fock space expansion of one-dimensional quantum gas with contact interaction with the transcorrelated method.
  Improvement from $\delta E \sim M^{-1}$ to $\delta E \sim M^{-3}$



P Jeszenszki, HJ Luo, A Alavi, JB, *arXiv:1806.032888*

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Thank you!