

Topology of (1D) Quantum Systems Out of Equilibrium

Nigel Cooper
Cavendish Laboratory, University of Cambridge

Controlling Quantum Matter: From Ultracold Atoms to Solids
Humboldt Kolleg, Vilnius, 30 July 2018

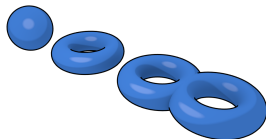
[Max McGinley & NRC, arXiv:1804.05756](#)



EPSRC

Engineering and Physical Sciences
Research Council

Topological Invariants



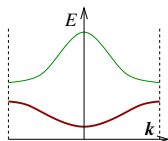
genus $g = 0, 1, 2, \dots$

$$\frac{1}{2\pi} \int_{\text{closed surface}} \kappa \, dA = (2 - 2g)$$

$$\text{Gaussian curvature } \kappa = \frac{1}{R_1 R_2}$$

2D Bloch bands

[Thouless, Kohmoto, Nightingale & den Nijs (1982)]



$$\text{Chern number } \nu = \frac{1}{2\pi} \int_{\text{BZ}} d^2k \, \Omega_k$$

$$\text{Berry curvature } \Omega_k = -i \nabla_k \times \langle u_k | \nabla_k u_k \rangle \cdot \hat{z}$$

- ν cannot change under smooth deformations of the energy band
- bulk insulator with ν (chiral) metallic surface states



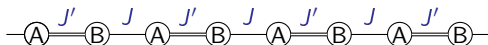
Topological Insulators

[Hasan & Kane, RMP 2010]

Many generalizations when *symmetries* included:

“symmetry-protected” topological insulators/superconductors

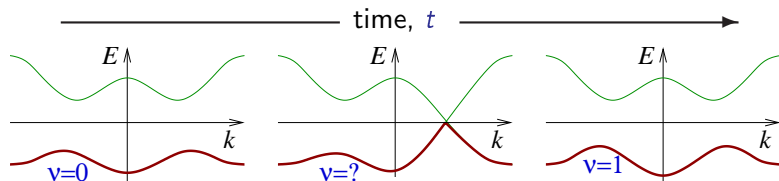
- Time-reversal symmetry (non-magnetic material in $B = 0$)
 $\Rightarrow$ 3D bulk insulator with metallic 2D surfaces
- Su-Schrieffer-Heeger model



$$H_k = - \begin{pmatrix} 0 & J' + J e^{-ika} \\ J' + J e^{ika} & 0 \end{pmatrix} \quad \begin{array}{l} \text{“chiral” symmetry} \\ \sigma_z H_k = -H_k \sigma_z \end{array}$$

$\Rightarrow$ 1D band insulator with gapless edge modes

Dynamical changes in band topology?

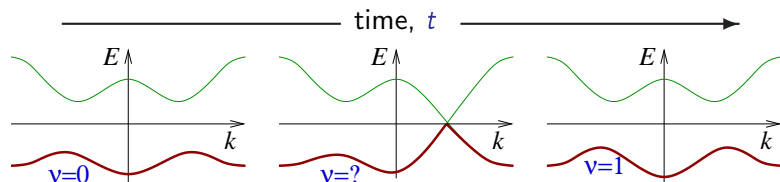


What are the consequences for the topology of the system?

- preparation of topological states?
- non-adiabatic $\Rightarrow$ meaning of topology out of equilibrium?

Dynamics of Chern Insulators (2D)

Quench: start in ground state of $\hat{H}^i$, time evolve with $\hat{H}^f$



Time-evolving Bloch state of fermion at k

$$|u_k(t)\rangle = \exp(-i\hat{H}_k^f t)|u_k(0)\rangle$$

$$\Omega_k(t) = -i\nabla_k \times \langle u_k(t)|\nabla_k u_k(t)\rangle \cdot \hat{z}$$

$\Rightarrow$ Chern number ν of the *state* is invariant

[L. D'Alessio & M. Rigol, Nat. Commun. (2015); M.D. Caio, NRC & M.J. Bhaseen, PRL (2015)]

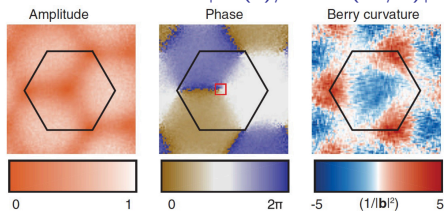
[“topological invariant” under smooth changes of the Bloch states]

Dynamics of Chern Insulators: Physical Consequences?

- Obstruction to preparation of state with differing Chern number
[For slow ramps, and in finite systems, deviations can be small.]

- Chern number can be obtained by tomography of Bloch states

Two-band model: $|u_{\mathbf{k}}(t)\rangle = \sin(\theta_{\mathbf{k}}/2)|A, \mathbf{k}\rangle - \cos(\theta_{\mathbf{k}}/2)\exp(i\phi_{\mathbf{k}})|B, \mathbf{k}\rangle$



[N. Fläschner, B. S. Rem, M. Tarnowski,
D. Vogel, D.-S. Lühmann, K. Sengstock
& C. Weitenberg, Science (2016)]

- Topology of *final* Hamiltonian can be uncovered by tracking the time evolution of the Bloch states in $(\mathbf{k}, t)$ space [Next talk, Xiong-Jun Liu]

Any differences for symmetry-protected topological classes?

Topology of 1D Quantum Systems (Free Fermions)

In 1D, all topological invariants can be determined by:

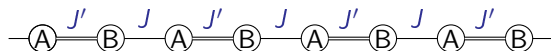
$$CS_1 = \frac{i}{2\pi} \int_{\text{BZ}} dk \langle u_k | \partial_k u_k \rangle$$

Equivalently: Berry phase around the Brillouin Zone (Zak phase)

Only quantized in the presence of symmetries

In 1D, topology must be protected by symmetry

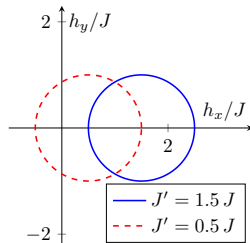
Example: Su-Schrieffer-Heeger Model



$$H_k = - \begin{pmatrix} 0 & J' + J e^{-ika} \\ J' + J e^{ika} & 0 \end{pmatrix} = -\mathbf{h}(k) \cdot \boldsymbol{\sigma}$$

Chiral symmetry $\Rightarrow \mathbf{h} = (h_x, h_y, 0) \Rightarrow h_x + i h_y \equiv |\mathbf{h}(k)| e^{i\phi(k)}$

$$CS_1 = \frac{1}{2} \underbrace{\frac{1}{2\pi} \int_{\text{BZ}} \frac{d\phi}{dk} dk}_{\text{integer}}$$



Is this topological invariant preserved out of equilibrium?

No... need to consider symmetries!

Symmetry-Protected Topology Out of Equilibrium

[Max McGinley & NRC, arXiv:1804.05756]

- Start in ground state of $\hat{\mathcal{H}}^i$, then time evolve with $\hat{\mathcal{H}}^f$
- $\hat{\mathcal{H}}^f$ breaks symmetry $\rightarrow$ topological “invariant” can vary
[“explicit symmetry breaking”]
- What if $\hat{\mathcal{H}}^f$ respects symmetry?

Symmetry can still be broken after a quench

▷ Antiunitary symmetries $[\langle \hat{\mathcal{O}}\Phi, \hat{\mathcal{O}}\Psi \rangle = \langle \Phi, \Psi \rangle^*]$

$$\hat{\mathcal{O}}e^{-i\hat{\mathcal{H}}t}\hat{\mathcal{O}}^{-1} = e^{+i\hat{\mathcal{H}}t}$$

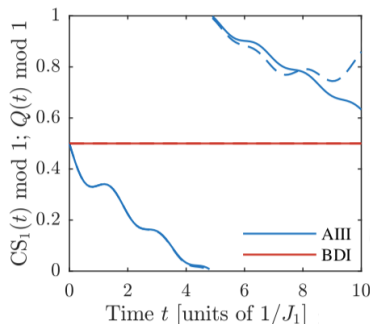
Symmetry broken in the non-equilibrium state $|\Psi(t)\rangle$
[“dynamically induced symmetry breaking”]

Topological “invariant” time-varying even if symmetries respected!

Time-Varying $CS_1(t)$: Physical Consequences

- Could be observed in Bloch state tomography [cf. Chern number]
- Directly measure, via: $\frac{d}{dt}CS_1(t) = j(t) = \dot{Q}(t)$
[cf. in 2D, Chern number $\neq$ Hall conductance out of equilibrium]

Example: quenches in a generalized SSH model



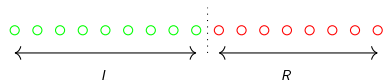
BDI: time-reversal, particle-hole & chiral

AIII: chiral symmetry only

Topological Classification Out of Equilibrium

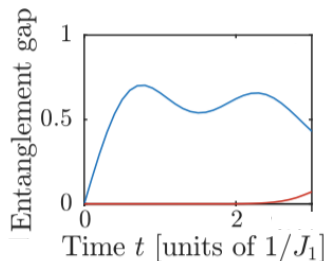
Equilibrium topological state $\Rightarrow$ gapless edge state

Non-equilibrium topological state $\Rightarrow$ gapless *entanglement spectrum*



$$|\Psi(t)\rangle = \sum_i e^{-\lambda_i} |\psi_L^i\rangle \otimes |\psi_R^i\rangle$$

Example: quenches in a generalized SSH model



BDI: time-reversal, particle-hole & chiral

AIII: chiral symmetry only

$\Rightarrow$ Meaningful topological classification out of equilibrium

Topological Classification Out of Equilibrium

“Ten-fold way” for free fermions

[Chiu, Teo, Schnyder & Ryu, RMP (2016)]

[Time-reversal, particle-hole, and chiral symmetries]

Non-equilibrium classification in 1D

[Max McGinley & NRC, arXiv:1804.05756]

Class	T	C	S	$CS_1(t=0)$	$CS_1(t) \bmod 1$	eq. $\rightarrow$ non-eq.
AIII	0	0	1	$\mathbb{Z}/2^*$	Varies $[0, 1)$	$\mathbb{Z} \rightarrow 0$
BDI	+	+	1	$\mathbb{Z}/2^*$	Const. $\{0, 1/2\}$	$\mathbb{Z} \rightarrow \mathbb{Z}_2$
D	0	+	0	$\mathbb{Z}/2 \bmod 1$	Const. $\{0, 1/2\}$	$\mathbb{Z}_2 \rightarrow \mathbb{Z}_2$
DIII	−	+	1	$\mathbb{Z} \bmod 2^*$	Const. 0	$\mathbb{Z} \rightarrow 0$
CII	−	−	1	$\mathbb{Z}^*$	Const. 0	$\mathbb{Z} \rightarrow 0$

- Out of equilibrium, topological “invariants” can vary in time:
“dynamically induced symmetry breaking”
⇒ no obstruction to changing the topology of the state dynamically
⇒ sensitivity to noise
- In 1D such time-variations appear as a measurable current
- There is a robust topological classification of non-equilibrium states, which differs from that at equilibrium:
bulk-boundary correspondence applies to entanglement spectrum
[holds also for interacting + disordered systems]

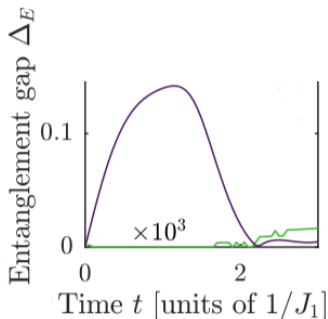
Generalization to Interacting SPT phases

[Max McGinley & NRC, arXiv:1804.05756]

Example: the Haldane phase of a $S = 1$ spin chain (e.g. AKLT model) is an SPT phase, stabilized by various symmetries:

- TRS (anti-unitary)
- $Z_2 \times Z_2$ dihedral symmetry (unitary)

Only unitary symmetries are preserved out of equilibrium



TRS only (purple)
both TRS and dihedral (green)