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Anomalous momentum diffusion

of strongly interacting bosons in optical lattices

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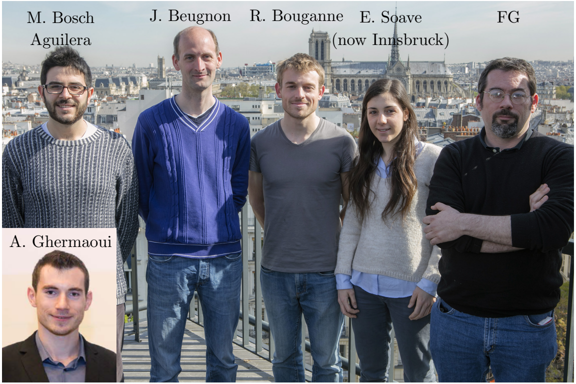
Humboldt Kolleg, Vilnius, August 1st 2018



Former members :

Q. Beaufils

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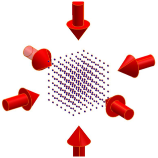
M. Scholl

E. Soave

Some recent works :

- *Revealing the Topology of Quasicrystals with a Diffraction Experiment*
  [Dareau *et al.*, Phys. Rev. Lett. **119**, 21530 (2017).
- *Clock spectroscopy of interacting bosons in deep optical lattices*
  [Bouganne *et al.*, New J. Phys. **19**, 113006 (2017).

Superfluid-Mott insulator transition for bosons

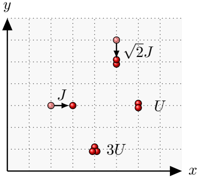


Optical lattices :

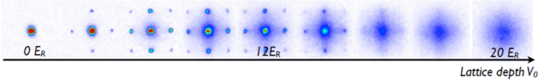
interference pattern can be used to trap atoms in a periodic structure

Bosons in the Bose-Hubbard regime :

1. Quantum tunneling favor delocalization
2. **Repulsive** on-site interactions favor localization



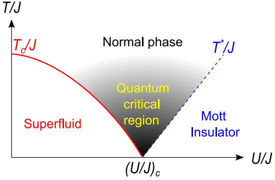
Quantum phase transition from a superfluid, Bose-condensed ground state to a Mott insulator



Greiner *et al.*, Nature 2002.

Energy/Temperature scales : nanoKelvin

Time scales $\sim 10\,$ms



Time-of-flight interferences :

essentially a free flight revealing the *momentum distribution*

$$n_{\text{tof}}(\boldsymbol{r}, r) \approx n\left(\boldsymbol{K} = \frac{M\boldsymbol{r}}{\hbar t}\right) = \mathcal{G}\left(\boldsymbol{K}\right)\mathcal{S}\left(\boldsymbol{K}\right)$$

- smooth “Wannier” enveloppe $\mathcal{G}(\boldsymbol{K})$
- structure factor $\mathcal{S}(\mathbf{K}) = \sum_{i,j} e^{i\mathbf{K}\cdot(\mathbf{r}_j - \cdot\mathbf{r}_i)}\langle \hat{a}_i^\dagger a_j\rangle$

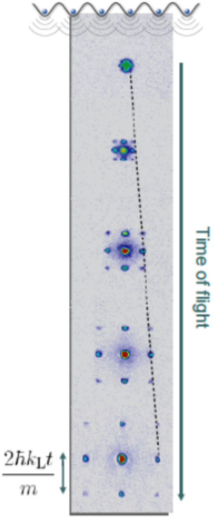
Single-particle density matrix : $\rho^{(1)}(\boldsymbol{r}, \boldsymbol{r}') = \langle \hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r}')\rangle$

Contrast of interference fringes when two matter waves overlap.

Lattice version : $\mathcal{C}(i, j) = \langle \hat{a}_i^\dagger \hat{a}_j\rangle$

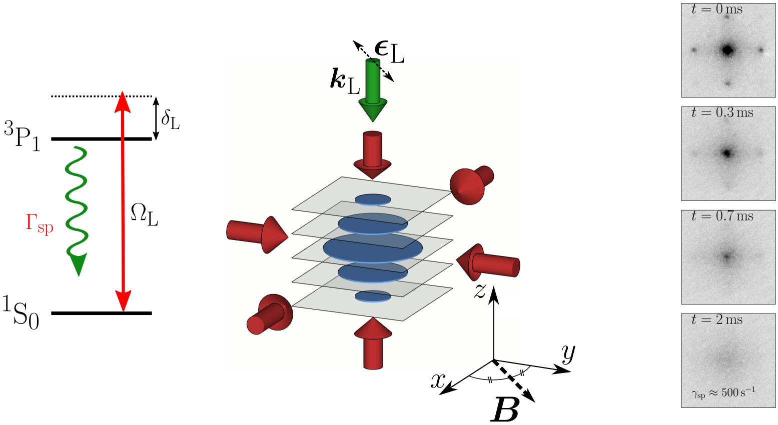
Momentum distribution $n(\boldsymbol{k})$: Fourier transform of $\rho^{(1)}$

Lattice version : $\mathcal{S}(\mathbf{K})$



Quantum-degenerate ^{174}Yb atoms in a 3D optical lattice

How does spontaneous emission destroy the spatial coherences initially present in the superfluid ?

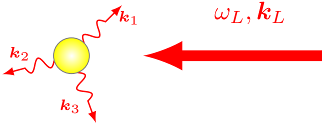


Absorption/spontaneous emission cycles with rate γ_{sp}

Momentum diffusion in resonant atom-light interaction

Momentum diffusion :

- Random momentum kicks after SE
- Random walk in momentum space:
  $\Delta k = \sqrt{2Dt}$ D : Diffusion coefficient
- Central role in laser cooling : limit temperature $T \propto \frac{D}{\text{friction}}$



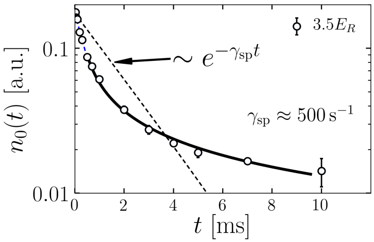
Equivalent point of view : destruction of spatial coherences $\rho^{(1)}(\boldsymbol{r}, \boldsymbol{r}')$

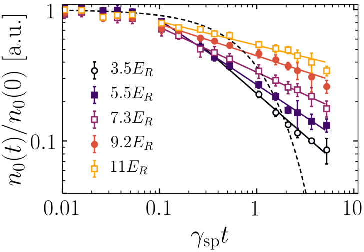
- Spatial correlations beyond $\lambda_0/(2\pi)$ strongly suppressed [Pfau *et al.*, PRL 1994]
- Exponential decay in time for given $\boldsymbol{r}, \boldsymbol{r}'$
- Interpretation : Modern version of Heisenberg's microscope

  continuous, weak measurements of the atom position [Marsteiner *et al.*, PRL 1996]

Anomalous momentum diffusion

Basic analysis : monitor $n_{\boldsymbol{k}=0} \equiv n_0$ (proxy for condensed fraction)



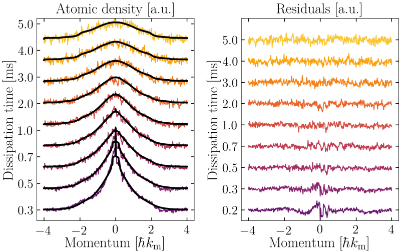


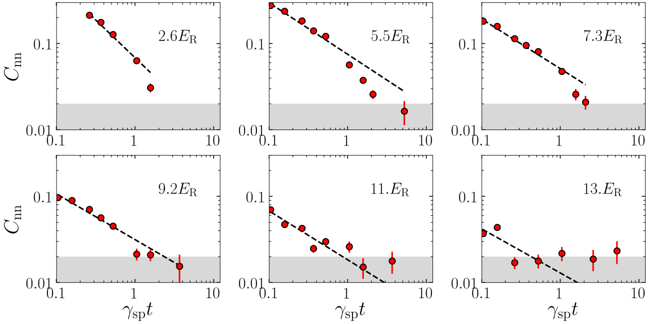
- Exponential (linear ...) decay at short times $t \leq t_{\rm cross}$
- Algebraic decay at long times: $\Delta k \sim t^{\alpha}$, $n_{\boldsymbol{k}=0} \sim 1/t^{2\alpha}$ with $\alpha < 1/2$
- Normal momentum diffusion : exponential decay

$$S_0(\boldsymbol{k}) = \sum_{\boldsymbol{R}\in\mathbb{Z}^2} \mathcal{C}_{\boldsymbol{R}} e^{i\boldsymbol{k}\cdot\boldsymbol{R}}$$
$$\approx 1 + \mathcal{C}_{\rm nn}\Big(\cos(k_x d) + \cos(k_y d)\Big) + \cdots$$

Nearest-neighbor correlation function :

$$\mathcal{C}_{\rm nn} = \sum_{\boldsymbol{r}_i} \langle \hat{a}^\dagger_{\boldsymbol{r}_i} \hat{a}_{\boldsymbol{r}_i+\boldsymbol{e}_x} \rangle$$



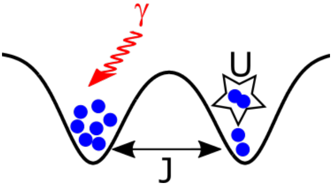


Dissipative Bose-Hubbard model :

$$\frac{d}{dt}\hat{\rho} = \frac{1}{i\hbar}\left[\hat{H}, \hat{\rho}\right] - \gamma\hat{\mathcal{L}}\left[\hat{\rho}\right], \qquad \hat{\mathcal{L}}\left[\hat{\rho}\right] = \sum_i \hat{n}_i\hat{\rho}\hat{n}_i - \frac{1}{2}\hat{n}_i^2\hat{\rho} - \frac{1}{2}\hat{\rho}\hat{n}_i^2.$$

Poletti *et al.*, PRL 2012, PRL 2013 (Kollath/Georges group)

See also Pichler *et al.*, PRA 2010 (Zoller group), Yanay and Mueller, PRA 2012



N bosons in two wells L, R.

Fock basis : $|n\rangle = |n_L = n, n_R = N - n\rangle$

Populations : $\rho_{n,n} = \langle n|\hat{\rho}|n\rangle$

Coherences : $\rho_{m,n} = \langle m|\hat{\rho}|n\rangle, m \neq n$

- Fock states are pointer states: $\langle n|\hat{\mathcal{L}}[\hat{\rho}]|n\rangle = 0$
- Coherences decay : $\langle m|\hat{\mathcal{L}}[\hat{\rho}]|n\rangle = -\frac{1}{2}\left(n - m\right)^2 \rho_{m,n}$
- Role of tunneling : partial restoration of short-range spatial coherence

$\Longrightarrow$ allows relaxation of populations

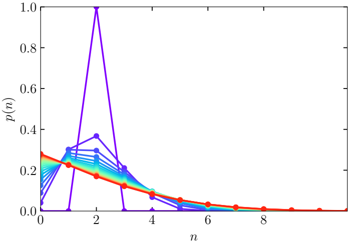
Effective Pauli master equation in the decoherent regime

Adiabatic elimination of fast variables (coherences)

Effective master equation for the probability p_n to find n atoms per site ($\Delta t \gg \gamma^{-1}$) :

$$\dot{p}_n \equiv \frac{\Delta p_n}{\Delta t} = W_{n+1}p_{n+1} + W_{n-1}p_{n-1} - 2W_n p_n$$

Poletti *et al.*, PRL 2012, PRL 2013



Three successive stages in the relaxation :

1. $t \lesssim \gamma^{-1}$: initial relaxation of coherences (not described by the master equation),
2. $\gamma^{-1} < \gamma t \ll t^*$: algebraic regime with slow decay of populations,
3. $t \gtrsim t^*$: final relaxation to the (infite temperature) steady state.

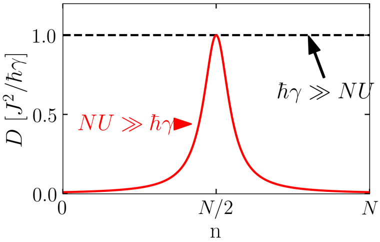
From regular to anomalous diffusion

Mapping to a Fokker-Planck equation for $N \gg 1$: $n \to x = \frac{n - \frac{N}{2}}{N}$, $p_n \to Np(x)$

$$\frac{\partial p(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[D(x)\frac{\partial}{\partial x}p(x,t)\right]$$

Scaling solution: $p(x) = \frac{1}{\tau^\beta} f\left(u = \frac{x}{\tau^\beta}\right)$

If $D \sim x^{-\eta}$, scaling exponent $\beta = \frac{1}{2+\eta}$



$\hbar\gamma \gg NU$: $D \approx \frac{2J^2}{\hbar^2\gamma}$

- "Quantum Zeno effect"
  Patil, Chakram, Vengalattore, PRL 2015
- D uniform :
  regular diffusion
- Scaling exponent $\beta = 1/2$

$\hbar\gamma \ll NU$: $D \approx \frac{J^2\gamma}{(NU)^2} \times \frac{1}{x^2}$

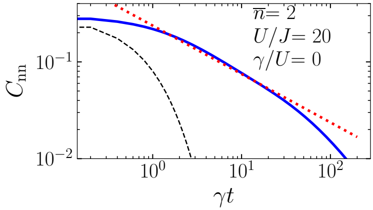
- "Interaction-induced impeding of decoherence"
- Power-law tail :
  Anomalous (sub-)diffusion
- Scaling exponent $\beta = 1/4$
  Poletti *et al.*, PRL 2012, PRL 2013

Coherence decay exponents versus lattice depth $V_\perp$

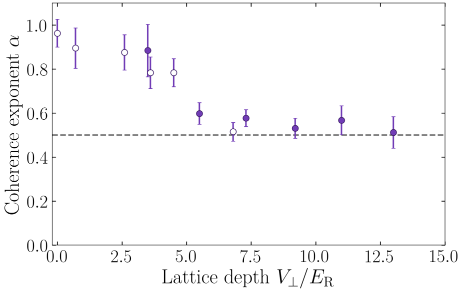
Phase coherence, scaling regime :

$$C_{\rm nn} = \langle \hat{a}^\dagger_{i\pm1} \hat{a}_i \rangle \propto \frac{1}{t^{2\beta}}$$

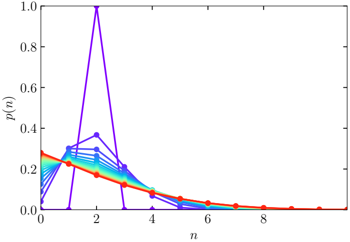
$$C_{\rm nn} \approx \frac{c_0}{\sqrt{2z\gamma t}} \text{ if } \begin{cases} \gamma \to 0 \\ \overline{n} \to \infty \end{cases}$$

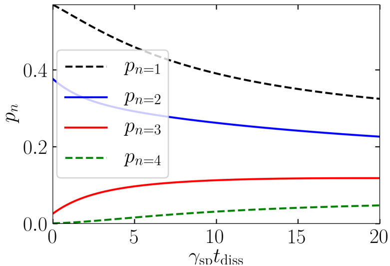


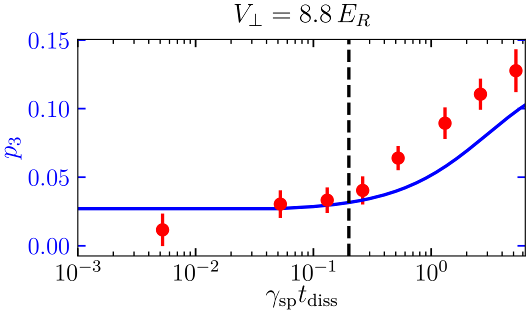
Comparison with experimental decay exponents of $C_{\rm nn}$:



Direct observation of Fock space dynamics using three-body losses







Conclusion

Decoherence of a bosonic many-body system under spontaneous emission

Main message : Interactions slow down decoherence

- Observation of anomalous momentum diffusion : $\Delta k \sim t^{1/4}$ instead of $\Delta k \sim \sqrt{t}$
- Interpretation as a signature of an underlying anomalous diffusion in Fock space
- Direct observation of Fock space dynamics using three-body losses

- Why is the "Poletti *at al.*" model working ?

  Many effects left out : dipole-dipole interactions, superradiance, interband transitions ...

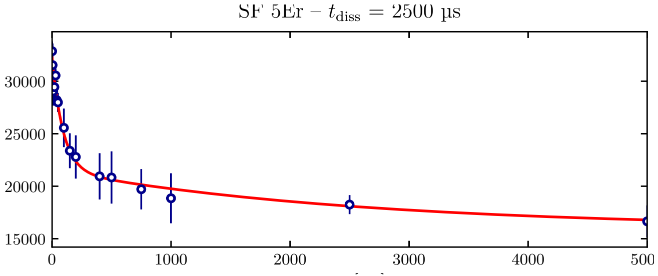
- Numerical study of XXZ chains [Cai and Barthel, PRL 2013] under dephasing :

  similar power-law slowdown of behavior as we observed (but not the Ising chain)

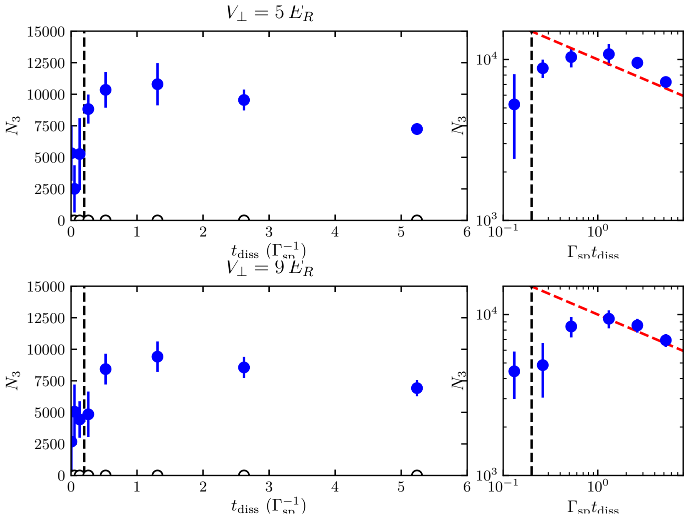
  Universality classes also relevant for non-equilibrium phenomena/decoherence ?

Loss dynamics after freezing

- Same experiment as before : illumination with near-resonant laser for t_{diss}
- then raise the horizontal lattice to “freeze” the density distribution
- wait for t_{hold} : three-body losses empty sites with $n \geq 3$
- monitor losses to extract $p(n \geq 3)$



Solid lines : fit with known loss time constants to extract $N_3 = 3p(3)$, etc ...


$V_\perp = 5\,E_R$
15000
12500
10000
7500
5000
2500
0
N_3
0 1 2 3 4 5 6
$t_{\text{diss}}\ (\Gamma_{\text{sp}}^{-1})$
10^4
10^3
10^{-1}
10^0
$\Gamma_{\text{sp}} t_{\text{diss}}$
$V_\perp = 9\,E_R$

Excited band populations

From fits to momentum profiles :