Non-equilibrium classification of topological quantum phases

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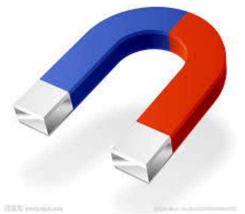
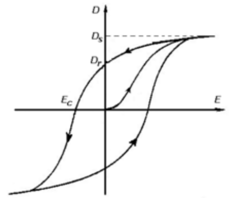
Humboldt Kolleg, Vilnius, 07/30/2018

Motivation

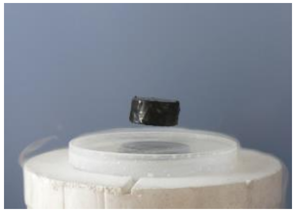
1. Classification of quantum phases: symmetry breaking phases vs topological phases

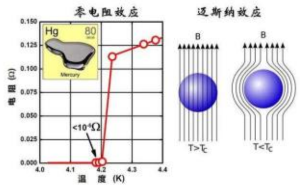
- Symmetry breaking phases: Landau-Ginzburg picture





Ferromagnet

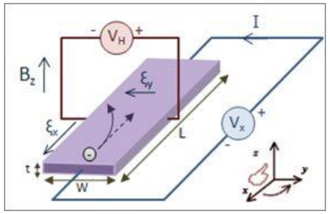


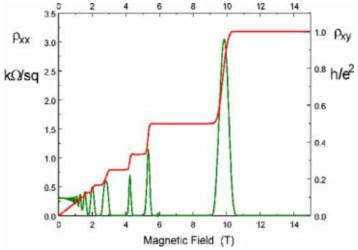


Superconductor

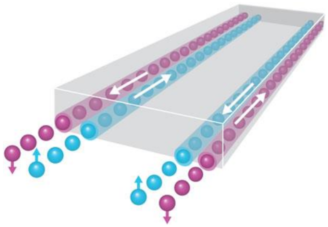
Characterization: local symmetry breaking orders (usually directly measurable)

- Topological quantum phases





Quantum Hall effect



Z_2 Topological insulator

Characterization: non-local topological invariants (characterize equilibrium ground states)

Challenges: 1) Usually not directly measurable; 2) Ground states are hard to prepare.

2. Motivation II: The mature technology of cold atoms allows to study non-equilibrium quantum dynamics of topological states.

- Naturally non-equilibrium
- Fully controllable
- Relatively long coherent time

→ Natural to study non-equilibrium quantum dynamics for cold atoms

A generic question: for a generic topological phase defined in equilibrium, can we find a non-equilibrium characterization of such phase?

? → Dynamical classification of topological quantum phases

Should be universal for a broad class of generic topological phases

Previous relevant works on quantum dynamics for topological phases: C. Wang etal., PRL 118, 185701 (2017); M. Tarnowski et al., arXiv:1709.01046; (HKUST+PKU) B. Song etal., Science Advances, 4, eeao4748 (2018). Valid for two-band models in specific dimensions.

The generic model

The goal: to establish dynamical classification for generic d-dim topological phases with integer (Z) invariants.

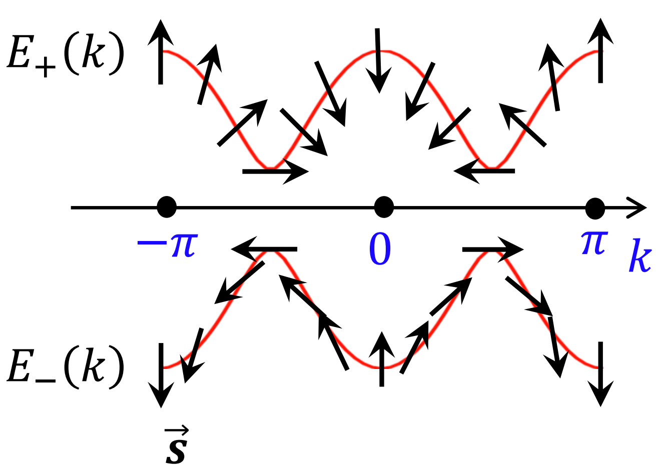
A few concrete examples

1. The 1D chiral topological phase: AIII class and Z invariant

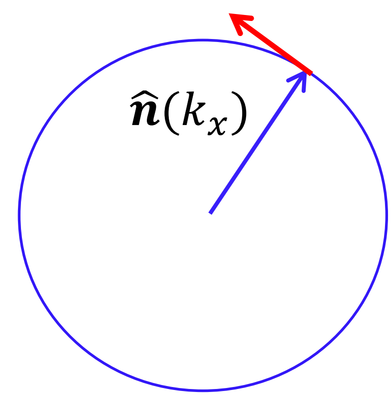
(Theory proposals: XJL, Z.-X. Liu, M. Cheng, PRL, 110, 076401 (2013); Pan, XJL*, Zhang*, Yi*, Guo, PRL 115, 045303 (2015); X. Zhou etal., PRL 119, 185701 (2017). Experiment: B. Song etal., Science Advances, 4, eeao4748 (2018))

$$\mathcal{H}_{\vec{k}} = (m_z - 2t_0 \cos k_x)\sigma_z + 2t_{so} \sin k_x \sigma_x$$

Topological spin texture in momentum space (for m_z=0)



Mapping: $k_x \mapsto \hat{\boldsymbol{n}}$



$$\hat{\boldsymbol{n}}(k_x) = \frac{\vec{s}}{|\vec{s}|}$$

$$\mathcal{N}_{1d} = \pm 1$$

Topology: classified by integer winding numbers: Z

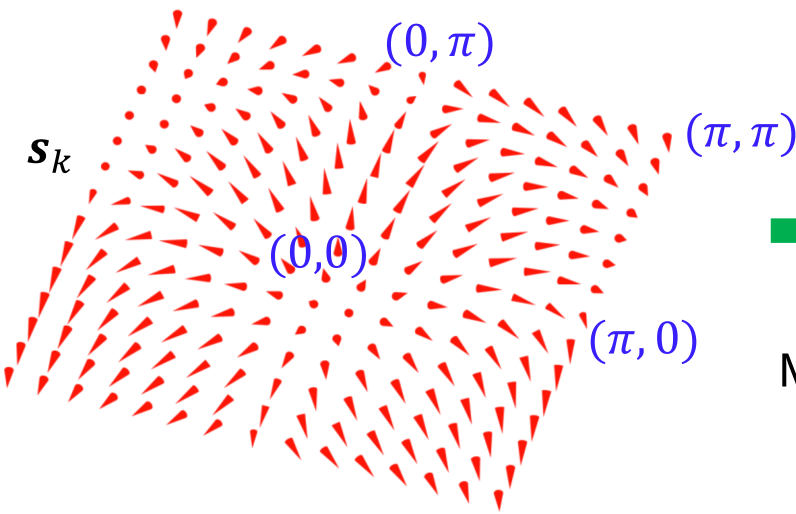
For a recent review: arXiv:1806.05628

2. 2D Chern insulator phase: Z Chern invariants

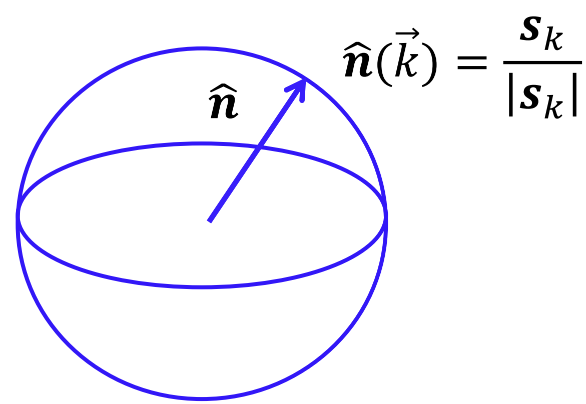
(Theory proposal: XJL, Law, Ng, PRL, **112**, 086401 (2014); PRL, 113, 059901 (2014); Theory proposal + Experiment (PKU+USTC): Wu etal., Science, **354**, 83 (2016)).

$$\mathcal{H}_{\vec{k}} = [m_z - 2t_0(\cos k_x + \cos k_y)]\sigma_z + 2t_{so}(\sin k_x \sigma_x + \sin k_y\, \sigma_y)$$

Topological spin texture in k space



Mapping: $\vec{k} \mapsto \hat{\boldsymbol{n}}$



Topology: classified by integer Chern numbers: Z

$$C_1 = \begin{cases} \mathrm{sgn}(m_z), & 0 < |m_z| < 4t_0; \\ 0, & |m_z| > 4t_0 \text{ or } m_z = 0; \end{cases} \quad \begin{matrix}\text{topological;} \\ \text{trivial.}\end{matrix}$$

For a recent review: arXiv:1806.05628

The generic model

Consider a generic d-dimensional gapped phase described by

$$\mathcal{H}(\mathbf{k}) = \vec{h}(\mathbf{k}) \cdot \vec{\gamma} = h_0(\mathbf{k})\gamma_0 + \sum_{i=1}^{d} h_i(\mathbf{k})\gamma_i$$

Examples:

For d=1, one can choose $\gamma_0 = \sigma_z, \gamma_1 = \sigma_x$.
For d=2, one can choose $\gamma_0 = \sigma_z, \gamma_1 = \sigma_x, \gamma_2 = \sigma_y$.
For d=3, one can choose $\gamma_0 = \sigma_z \otimes \rho_z, \gamma_1 = \sigma_x \otimes I, \gamma_2 = \sigma_y \otimes, \gamma_3 = \sigma_z \otimes \rho_x$
….

The topology is characterized by *Z* invariants

$$\nu_{2n-1} = \frac{(-1)^{n-1}(n-1)!}{2(2\pi \mathrm{i})^n (2n-1)!} \int_{\mathrm{BZ}} \mathrm{Tr}\left[\gamma \mathcal{H} (\mathrm{d}\mathcal{H})^{2n-1}\right]$$ d=2n-1 winding number

$$\mathrm{Ch}_n = -\frac{1}{2^{2n+1}} \frac{1}{n!} \left(\frac{\mathrm{i}}{2\pi}\right)^n \int_{\mathrm{BZ}} \mathrm{Tr}\left[\mathcal{H} (\mathrm{d}\mathcal{H})^{2n}\right]$$ d=2n n-th Chern number

Question: can we simplify the characterization of the bulk topology?

L. Zhang, L. Zhang, S. Niu, and XJL, 1802.10061v2

A key concept: band inversion surfaces

We decompose the $\vec{h}$-vector as two parts:

$$\vec{h} = (h_0, h_1, \dots, h_d) = (h_0, \boldsymbol{h}_{so}), \qquad \boldsymbol{h}_{so} = (h_1, h_2, \dots, h_d)$$

h_0: characterize band "dispersion"

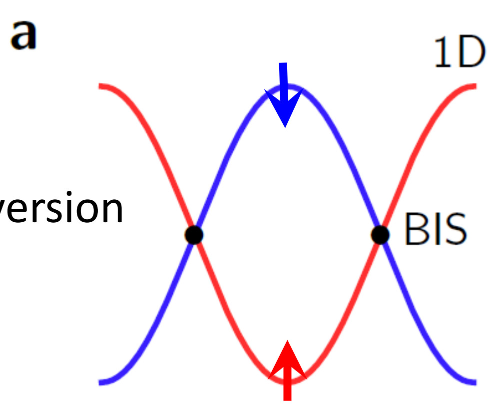
$\boldsymbol{h}_{so}$: characterize "spin-orbit" coupling

Band inversion surfaces (BISs) are (d-1)D space, defined by the momentum points with

$$h_0(\boldsymbol{k}) = 0; \quad \text{for} \quad \boldsymbol{k} \text{ at BISs.}$$

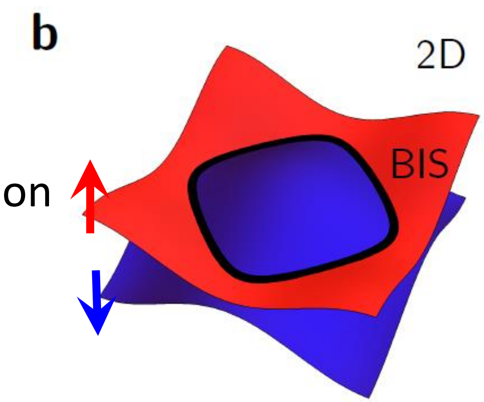
Examples (1D and 2D):

Band inversion points



$h_0(k)$=$m_z - 2t_s \cos k_x$

Band inversion ring

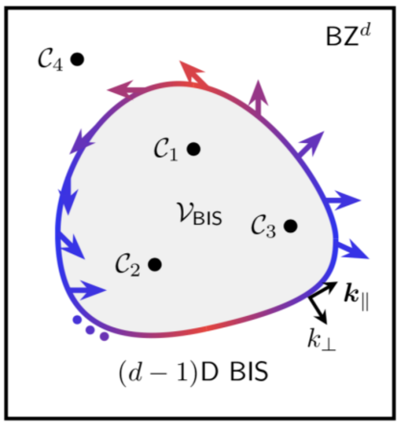


$$h_0(\boldsymbol{k}) = m_z - 2t_s(\cos k_x + \cos k_y)$$

Bulk-surface duality

Theorem I: the *d*-dimensional topological invariant (winding number or Chern number) can be reduced to a (*d*-1)-dimensional invariant defined on BISs

$$w_{d-1} = \frac{\Gamma(d/2)}{2\pi^{d/2}} \frac{1}{(d-1)!} \int_{\mathrm{BIS}} \hat{\boldsymbol{h}}_{\mathrm{so}} (\mathrm{d}\hat{\boldsymbol{h}}_{\mathrm{so}})^{d-1} \qquad \hat{\boldsymbol{h}}_{\mathrm{so}}(\mathbf{k}) = \boldsymbol{h}_{\mathrm{so}}/|\boldsymbol{h}_{\mathrm{so}}|$$



The (*d*-1)-dimensional invariant is indeed a summation of topological charges (at $\boldsymbol{h}_{so} = \mathbf{0}$) enclosed by the BISs.

$$w_{d-1} = \sum_{i \in \mathcal{V}_{\mathrm{BIS}}} \mathcal{C}_i,$$

The theorem implies that characterizing the topology of a d-dimensional topological phase can be mapped to the topological invariant on (d-1)D band inversions surfaces.

L. Zhang, L. Zhang, S. Niu, and XJL, 1802.10061v2

Further questions:

1. How to identify such band inversions surfaces (BISs) in an experiment?

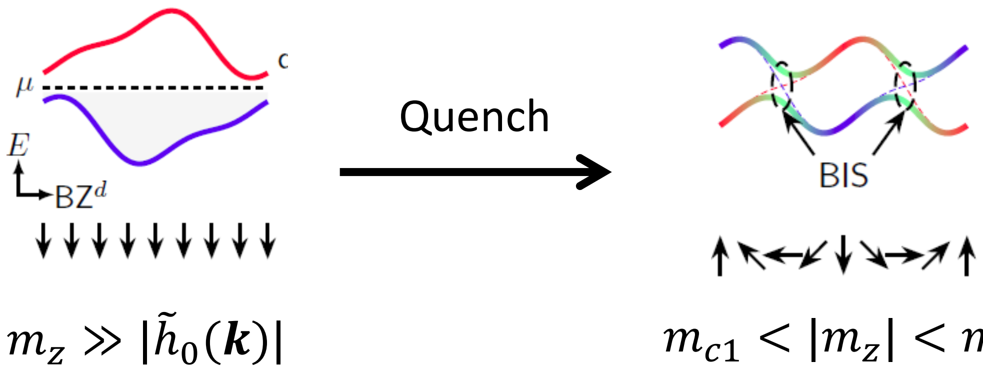
2. How to read out the information of topology from the BISs?

Quench dynamics

The idea: we consider the quench process from a deep trivial phase to topological phase.

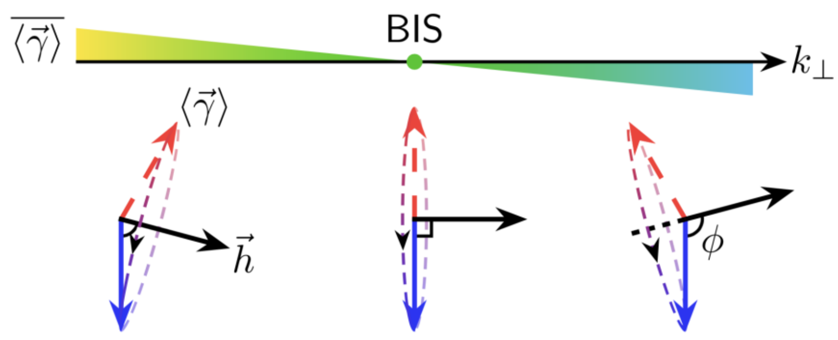
$$h_0 = \tilde{h}_0(\boldsymbol{k}) + m_z$$

Quench process:



The quantum dynamics

$$\overline{\langle \gamma_i \rangle} = \lim_{T\to\infty} \frac{1}{T} \int_0^T \mathrm{d}t \, \mathrm{Tr}\left[\rho e^{\mathrm{i}\mathcal{H}t} \gamma_i e^{-\mathrm{i}\mathcal{H}t}\right]$$



Spin procession

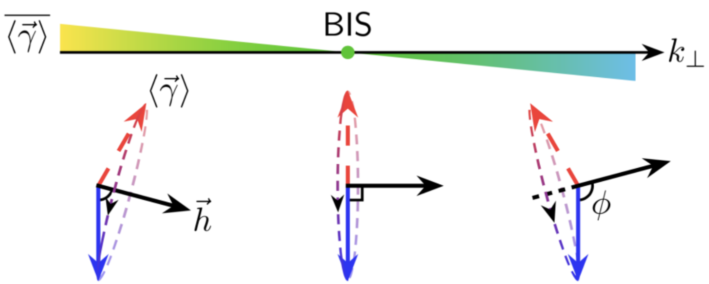
The results

Theorem II: the "band inversion surface" in the dynamical regime is characterized by

$$\overline{\langle\gamma_i\rangle} = 0 \qquad \text{For } i = 0,1,2,\ldots,d$$

Theorem III: the topological invariant of the d-dimensional system is eventually given by the dynamical quantity:

$$w_{d-1} = \frac{\Gamma(d/2)}{2\pi^{d/2}} \frac{1}{(d-1)!} \int_{\mathrm{BIS}} \widetilde{g(\boldsymbol{k})} \, (\mathrm{d}\, \widetilde{g(\boldsymbol{k})}\,)^{d-1}$$



Here a dynamical spin-texture field reads

$$\widetilde{\mathbf{g}(\mathbf{k})} \equiv -\frac{1}{\mathcal{N}_{\mathbf{k}}} \partial_{k_\perp} \overline{\langle\vec{\gamma}\rangle}$$

$k_\perp$ is perpendicular to band inversion surfaces.

L. Zhang, L. Zhang, S. Niu, and XJL, 1802.10061v2

Numerical simulations (for 2D)

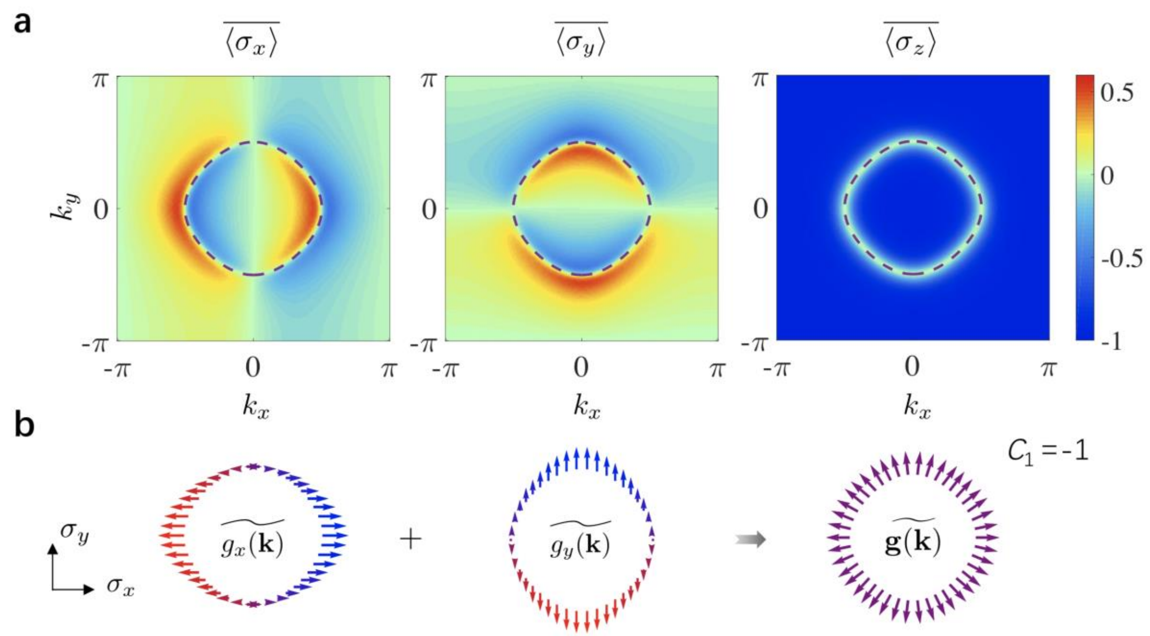
Application: the 2D Chern insulator (QAH) model (Theory: XJL, K. T. Law, and T. K. Ng, PRL, **112**, 086401 (2014); PRL, 113, 059901 (2014); Theory proposal + Experiment: Wu etal., Science, **354**, 83 (2016)).

$$\mathcal{H}_{\vec{q}} = \underbrace{[m_z - 2t_0(\cos q_x + \cos q_y)]}_{h_0(k)}\sigma_z + 2t_{so}(\sin q_x \sigma_x + \sin q_y\, \sigma_y)$$

Quench process: $m_z = 8t_0$ (trivial phase) $\longrightarrow$ $m_z = t_0$ (topological phase)

The time averaged spin dynamics

Dynamical topological spin texture:



L. Zhang, L. Zhang, S. Niu, and XJL, 1802.10061v2

Another quench process: $m_y = 50t_0$ $\longrightarrow$ $m_y = 0$

trivial phase → topological phase

$$\mathcal{H}_{\vec{q}} = [m_z - 2t_0(\cos k_x + \cos k_y)]\sigma_z + 2t_{so}\sin k_x \sigma_x + \underbrace{(m_y + 2t_{so}\sin k_y)}_{h_0(k)}\sigma_y$$

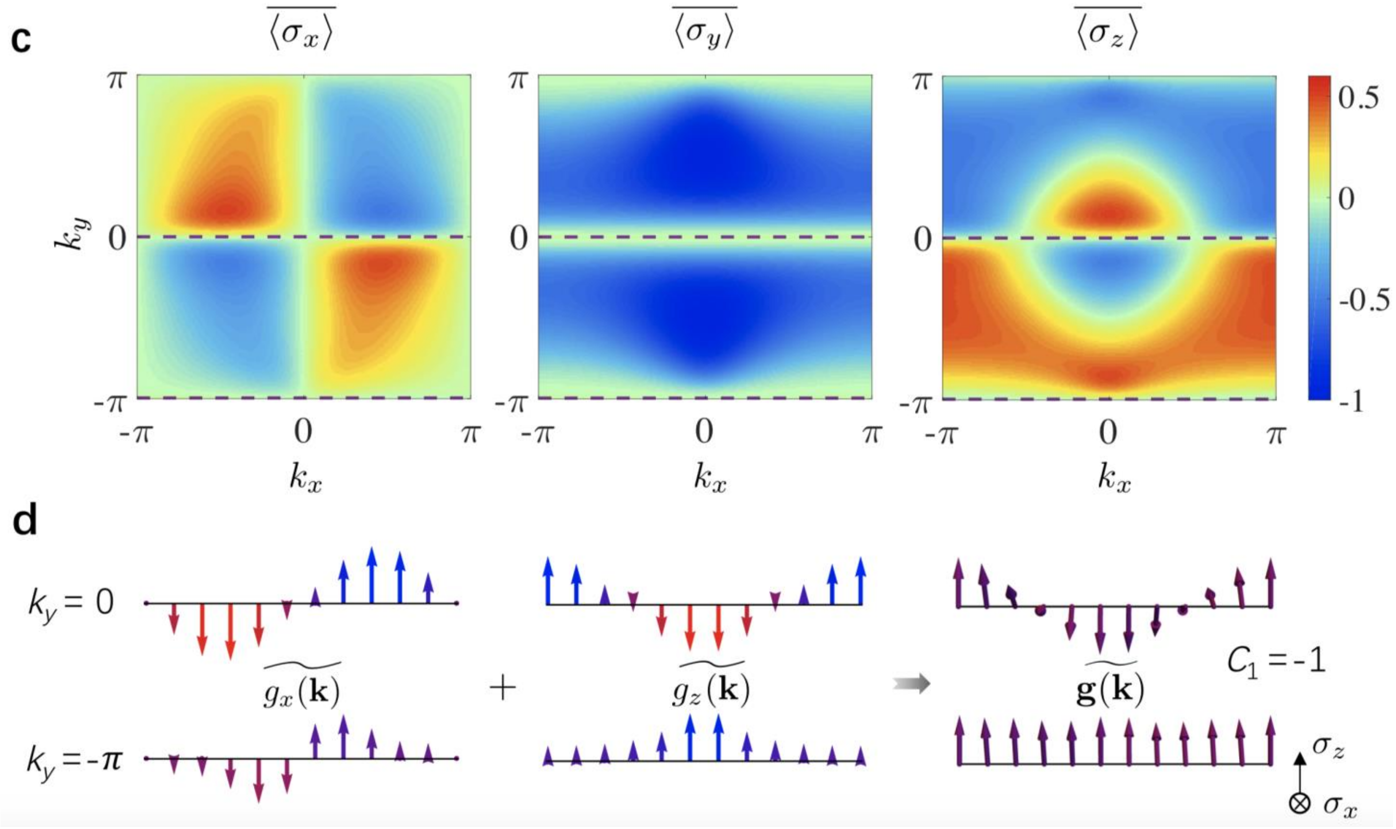
$h_0 \equiv h_y$

$\mathbf{h}_{\rm so} \equiv (h_x, h_z)$

while fixing $m_z = t_0$

The time averaged spin dynamics:

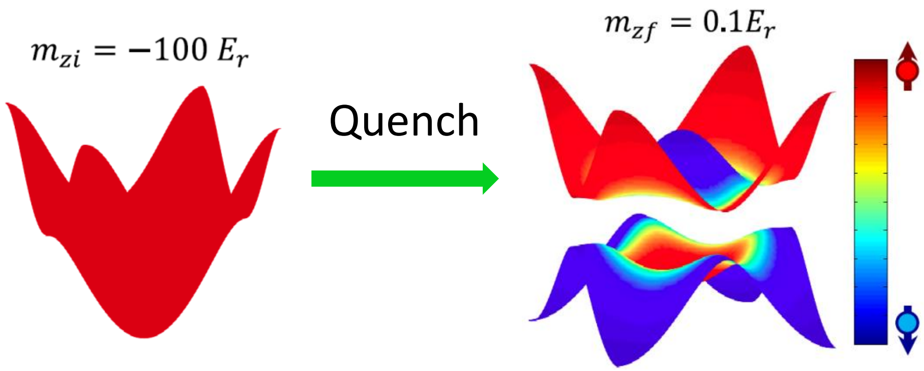
Dynamical topological spin texture:



L. Zhang, L. Zhang, S. Niu, and XJL, 1802.10061v2.

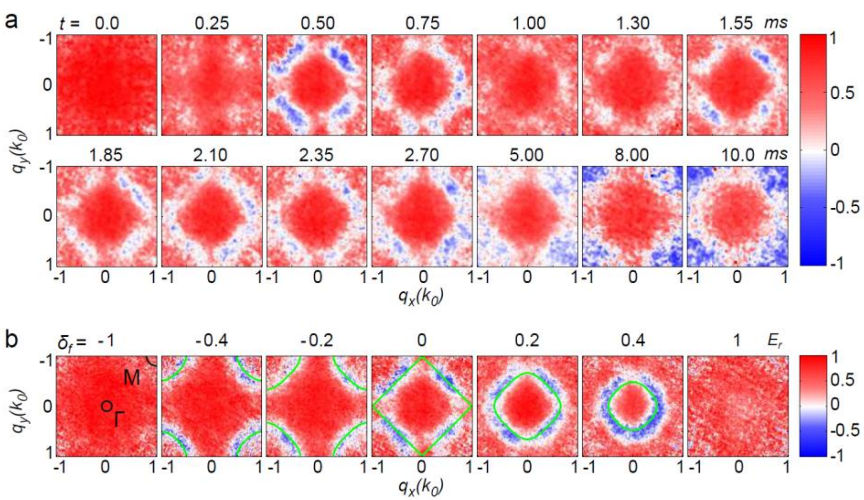
Experimental measurements

Quench experiment for 2D QAH model (XJL, Law, Ng, PRL, **112**, 086401 (2014); Wang, Lu, Sun, Chen, Deng, and XJL, PRA **97,** 011605 (R) (2018))

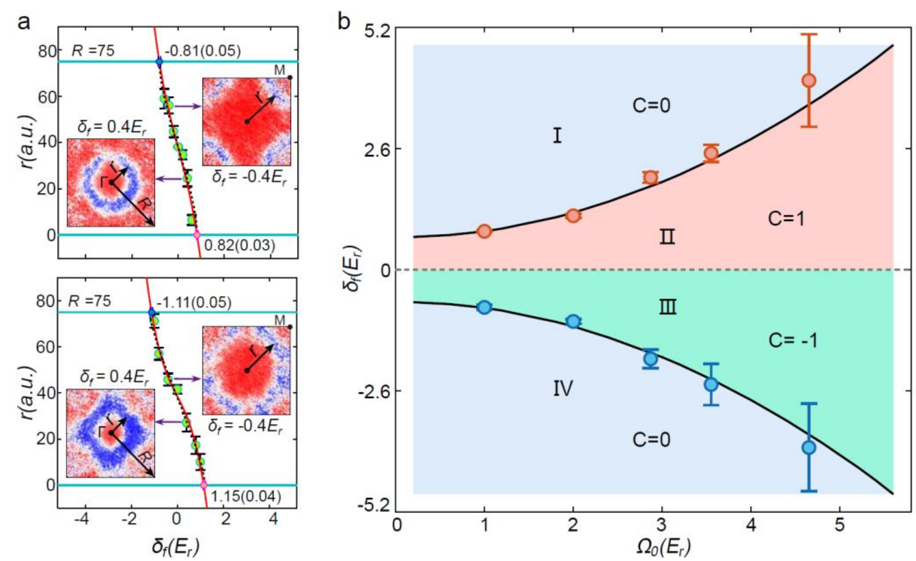


$$\mathcal{H}_{\vec{q}} = [m_z - 2t_0(\cos q_x + \cos q_y)]\sigma_z + 2t_{so}(\sin q_x \sigma_x + \sin q_y\, \sigma_y)$$

Spin dynamics (emergence of band inversion ring)



The measured phase diagram



W. Sun etal., Uncover topology by quantum quench dynamics, arXiv:1804. 08226.

Experimental measurements

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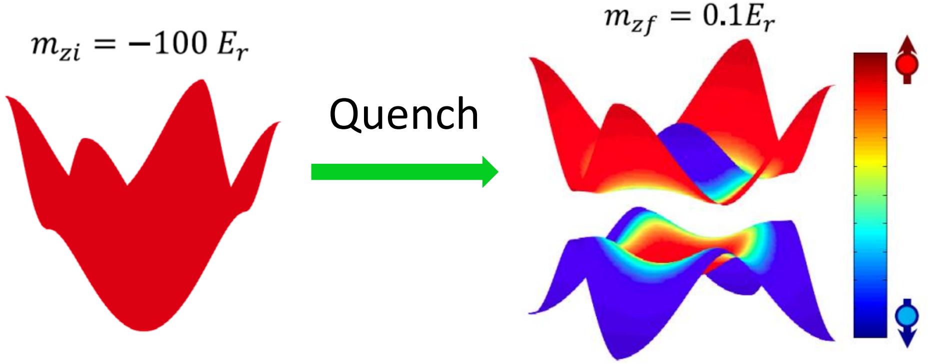
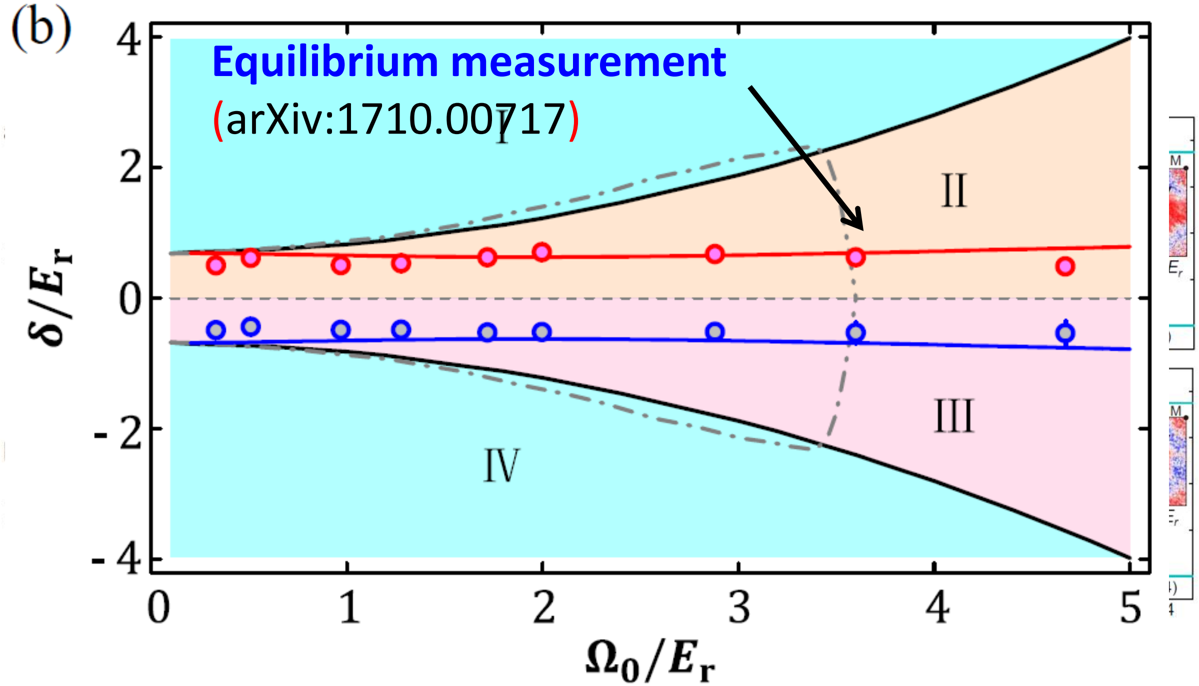
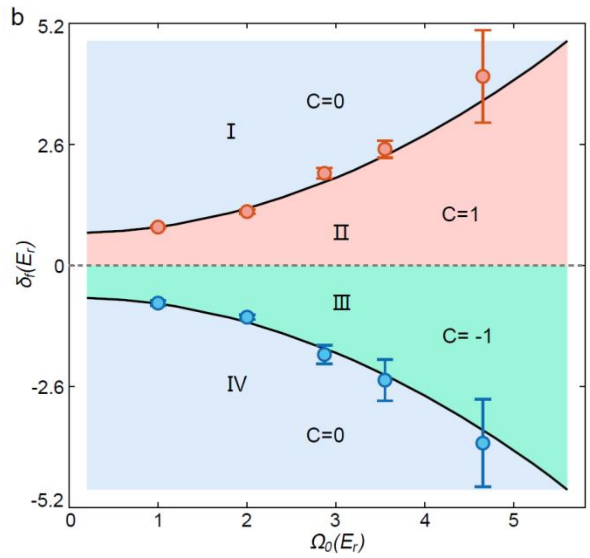


$$\mathcal{H}_{\vec{q}} = [m_z - 2t_0(\cos q_x + \cos q_y)]\sigma_z + 2t_{so}(\sin q_x \sigma_x + \sin q_y\, \sigma_y)$$

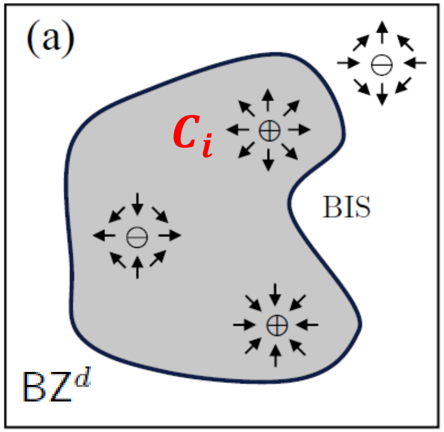


The measured phase diagram



W. Sun etal., Uncover topology by quantum quench dynamics, arXiv:1804. 08226.

Direct dynamical detection of topological charges

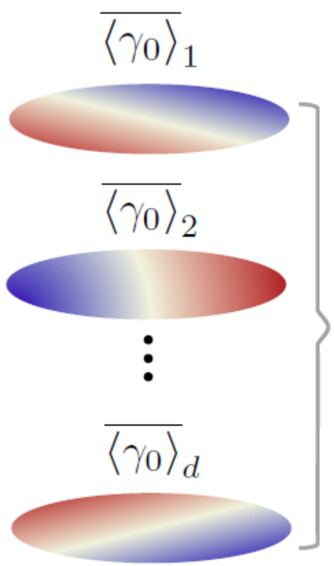


The (d-1)-dimensional invariant is indeed a summation of topological charges (at $\boldsymbol{h_{so}} = \mathbf{0}$) enclosed by the BIS.

$$w_{d-1} = \sum_{i \in \mathcal{V}_{\mathrm{BIS}}} \mathcal{C}_i,$$

Question: can we directly detect the topological charges?

A new quench scheme: a sequence of quench for all $\boldsymbol{h_i}$ axes, defining a dynamical vector field:



Quench $\boldsymbol{h_1}$ axies

Quench $\boldsymbol{h_2}$ axies

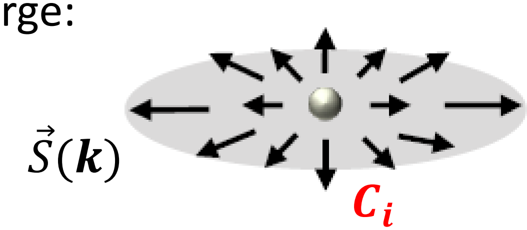
Quench $\boldsymbol{h_d}$ axies

Only measure $\overline{\langle\gamma_0\rangle}$

$$\vec{S}(\boldsymbol{k}) = (S_1, S_2, \ldots, S_d)$$

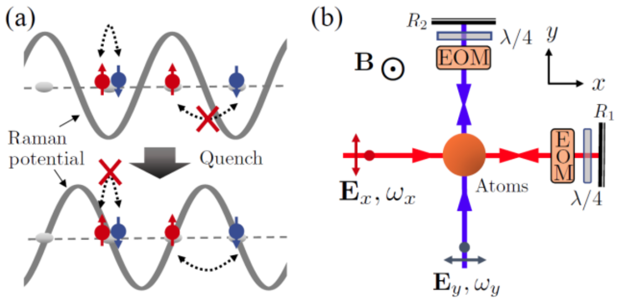
$$S_i\,(\boldsymbol{k}) \equiv -\frac{\mathrm{sgn}[h_0(\mathbf{k})]}{\mathcal{N}_{\mathbf{k}}} \overline{\langle\gamma_0(\mathbf{k})\rangle}_i$$

which characterize the topological charge:



Application to 2D QAH model

Application to 2D QAH model



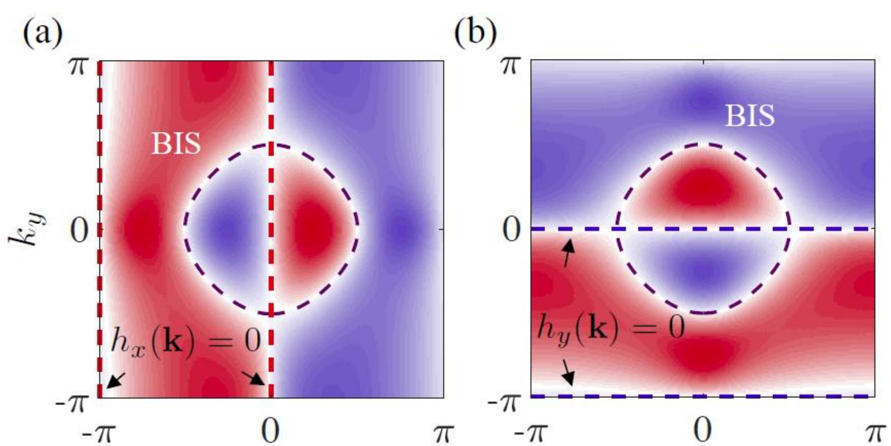
Quench processes:

$$\mathcal{H}_{\rm i} = [m_z - t_0(\cos q_x a + \cos q_y a)]\sigma_z + m_x\sigma_x + m_y\sigma_y$$

quench h_x, h_y

$$\mathcal{H}_{\rm f} = [m_z - t_0(\cos q_x a + \cos q_y a)]\sigma_z + t_{\rm so}\sin q_x a\,\sigma_y + t_{\rm so}\sin q_y a\,\sigma_x.$$

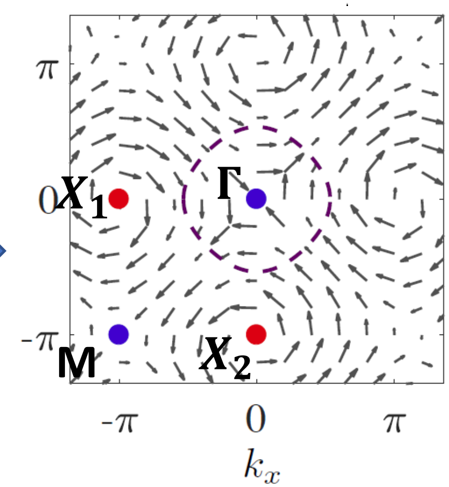
Spin dynamics for $\overline{\langle\sigma_z\rangle_i}$



quench h_x quench h_y

$\vec{S}(\boldsymbol{k})$ field

$m_z = t_0$:



Chern number = -1

Charges = -1 for **Γ** and **M** points

= +1 for X_1 and X_2 points

L. Zhang, L. Zhang, XJL, arXiv: 1807.10782(2018).

Summary

- Bulk-surface duality: d-dim topological phase ➡ (d-1)D band inversion surfaces (BISs).

- Dynamical bulk-surface correspondence I: BISs ➡ vanishing averaged spin polarizations in the quench dynamics.

- Dynamical bulk-surface correspondence II: the bulk topology ➡ dynamical topological invariant defined on the BISs.

- The dynamical bulk-surface correspondence can be directly measured in experiment, leading to the dynamical detection of topological phases with high precision.

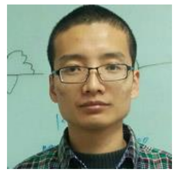
Theory: L. Zhang, L. Zhang, S. Niu, and XJL, 1802.10061v2.
L. Zhang, L. Zhang, and XJL, 1807.10782.
Experiments: W. Sun, …, J. Schmiedmayer, Y. Deng, XJL, S. Chen, and J. -W. Pan, 1804.08226.
HKUST (G. -B. Jo) & PKU, to appear soon.

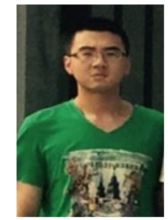
Acknowledgement

Group@PKU

Students





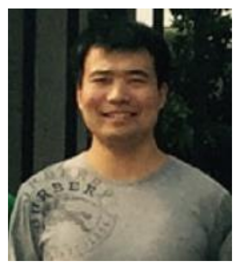




Yu-Qin Chen, Ying-Ping He, Xiang-Ru Kong, Sen Niu, Ting-Fung Jeffrey Poon, Bao-Zong Wang (USTC/PKU), Lin Zhang

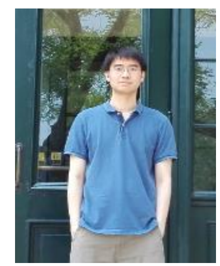
Yue-Hui Lu (undergraduate student)

Postdoctors

Dr. Long Zhang



Dr. Yu-Cheng Wang



Dr. Cheung Chan

Experimental groups

USTC group: Prof. Jian-Wei Pan, Prof. Shuai Chen, Prof. Youjin Deng

HKUST group: Prof. Gyu-Boong Jo

Thank you for your attention!

3. 3D chiral topological insulator: Z winding invariants

Appendix

$$\mathcal{H}(\mathbf{k}) = \vec{h}(\mathbf{k}) \cdot \vec{\gamma}$$

where $h_0(\mathbf{k}) = m_0 - t_0 \sum_j \cos(k_j) \quad h_j(\mathbf{k}) = t_{\rm so} \sin(k_j)$

γ matrices

$$\gamma_0 = \sigma_z \otimes \tau_x$$
$$\gamma_1 = \sigma_x \otimes \mathbf{1}$$
$$\gamma_2 = \sigma_y \otimes \mathbf{1}$$
$$\gamma_3 = \sigma_z \otimes \tau_z$$

obey the Clifford algebra $\{\gamma_i, \gamma_j\} = 2\delta_{ij}\mathbf{1}$

The chiral symmetry defined by $S = \sigma_z \otimes \tau_y$

$$S\mathcal{H}(\mathbf{k})S^{-1} = -\mathcal{H}(\mathbf{k})$$

AIII class

Topology: classified by integer 3D winding numbers: v_3

(I) $v_3 = -1$ for $t_0 < m_z < 3t_0$;
(II) $v_3 = 2$ for $-t_0 < m_z < t_0$;
(III) $v_3 = -1$ for $-3t_0 < m_z < -t_0$;
(IV) $v_3 = 0$ otherwise.

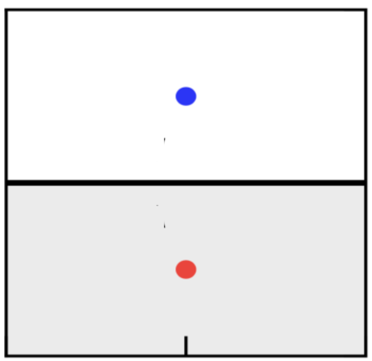
L. Zhang, L. Zhang, S. Niu, and XJL, 1802.10061v2

Numerical results

- 2D QAH model

$$h_0 \equiv h_y$$

$$\mathbf{h}_{\rm so} \equiv (h_x, h_z)$$

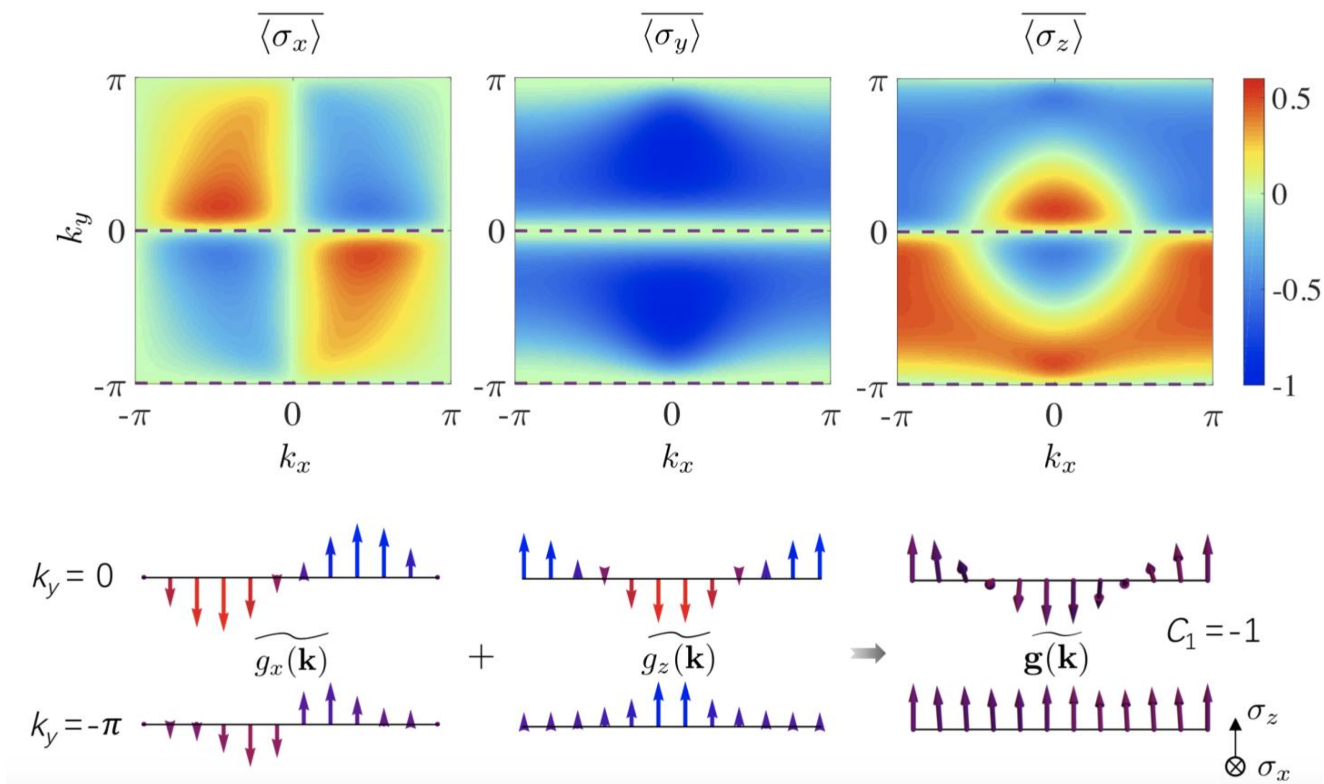


● charge -1

● charge +1

$m_y = 50t_0 \to 0$ while fixing $m_z = t_0$

$t^x_{\rm so} = 0.5 t^y_{\rm so} = t_0$

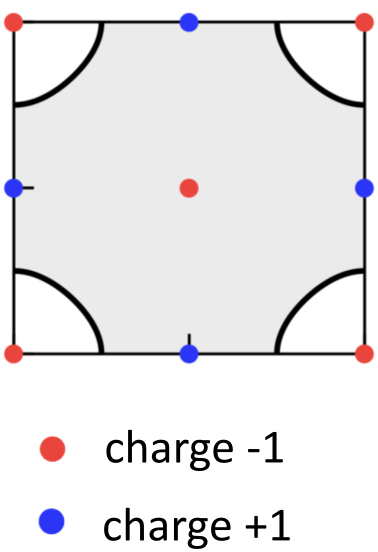


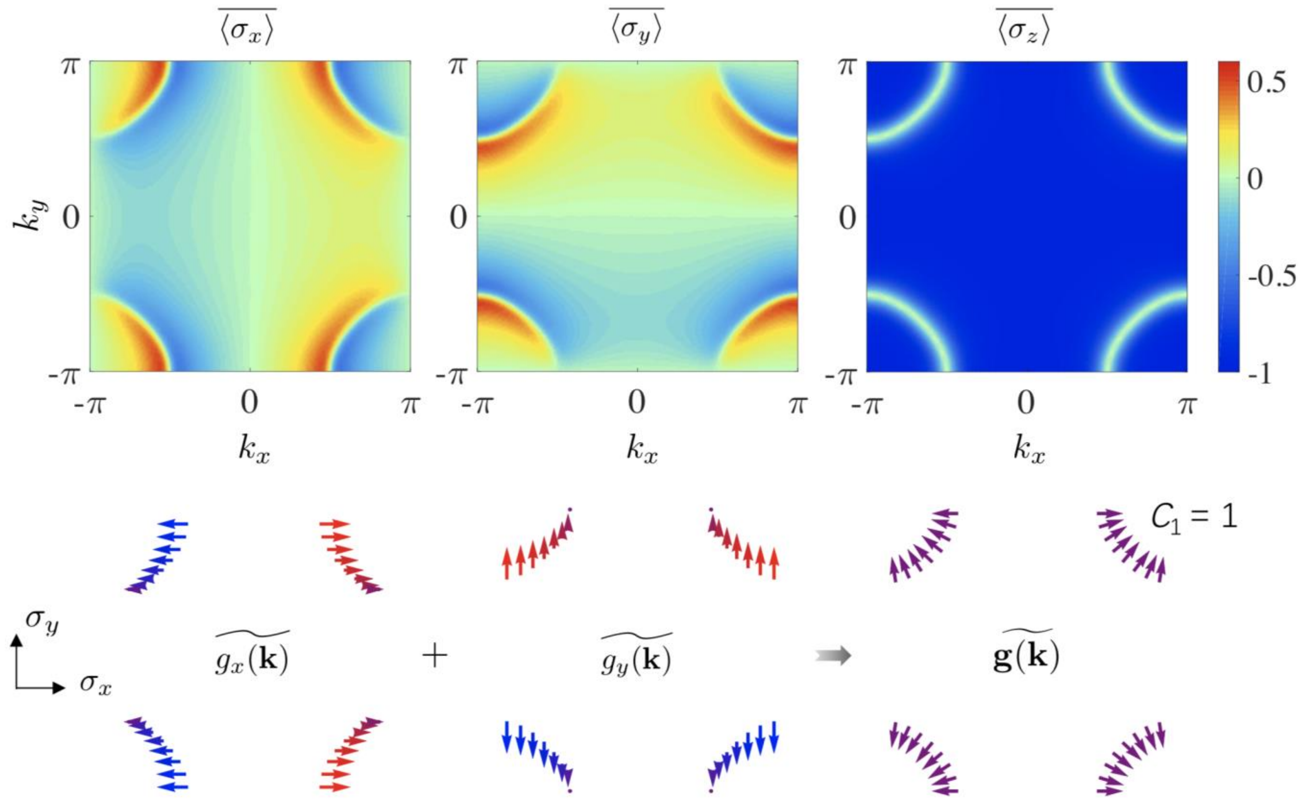
Another quench process: $m_z = 8t_0 \longrightarrow m_z = -t_0$

trivial phase → topological phase

$$\mathcal{H}_{\vec{q}} = \underbrace{[m_z - 2t_0(\cos q_x + \cos q_y)]}_{h_0(k)}\sigma_z + 2t_{so}(\sin q_x \sigma_x + \sin q_y\, \sigma_y)$$

The time averaging of spin dynamics





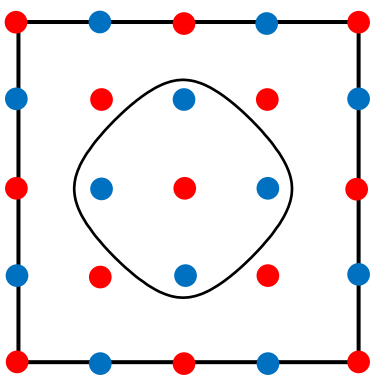
Numerical results

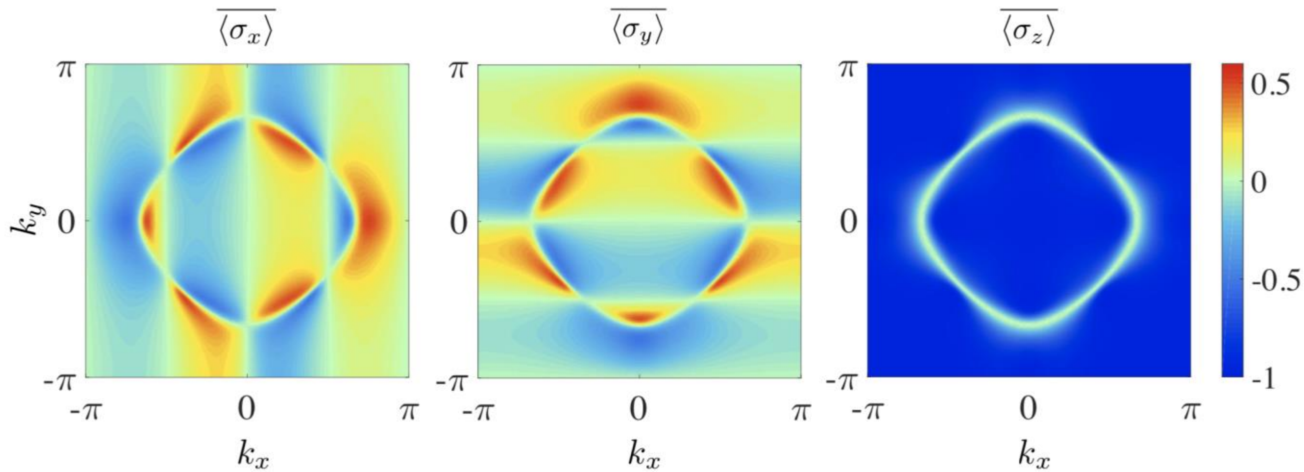
- Topological phases with high invariant

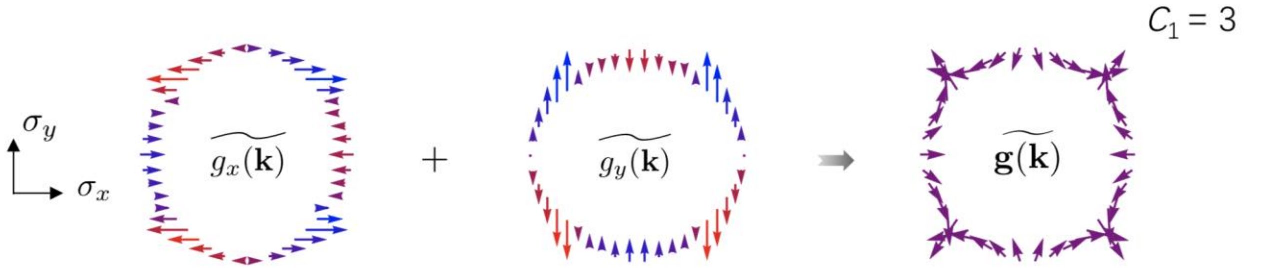
$\vec{h}_1(\mathbf{k}) = (t_{\rm so}\sin 2k_x, t_{\rm so}\sin 2k_y, m_z - t_0\cos k_x - t_0\cos k_y)$ $m_z = 8t_0 \to 0.5t_0$

Topological phases:

(i) $t_0 < m_z < 2t_0$, C_1 =−1;
(ii) $0<m_z <t_0$, C_1 =3;
(iii) $-t_0 <m_z <0$, C_1 =−3;
(iv) $-2t_0 <m_z <-t_0$, C_1 = 1.







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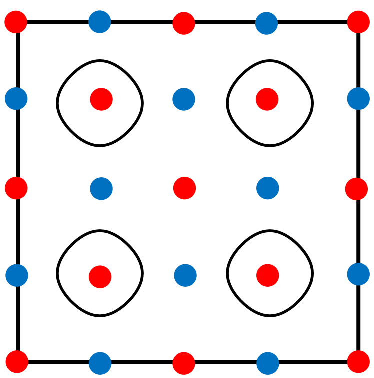
Numerical results

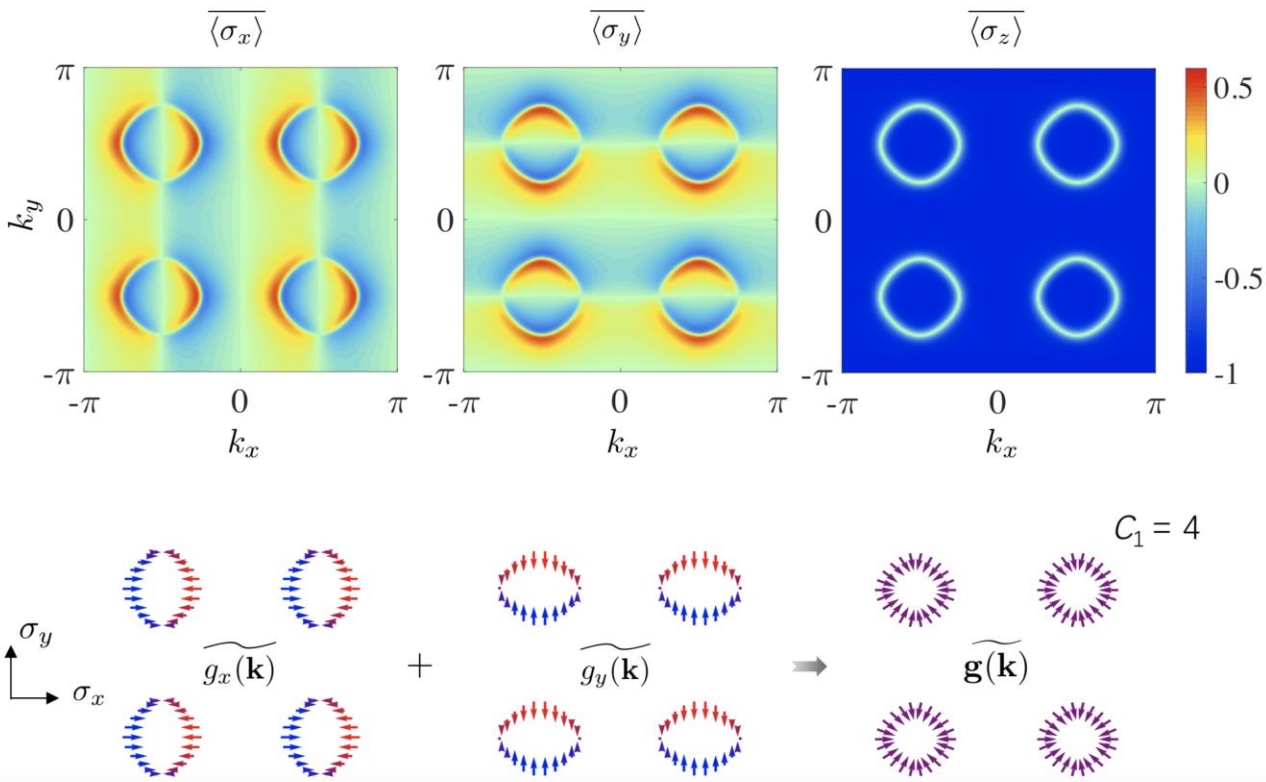
- Topological phases with high invariant

$$\vec{h}_2(\mathbf{k}) = (t_{\text{so}} \sin 2k_x, t_{\text{so}} \sin 2k_y, m_z - t_0 \cos 2k_x - t_0 \cos 2k_y) \qquad m_z = 8t_0 \to -t_0$$

Topological phases:

(i) $0 < m_z < 2t_0$, $C_1 = -4$;
(ii) $-2t_0 < m_z < 0$, $C_1 = 4$.





Numerical results

- 3D model

$$\mathcal{H}(\mathbf{k}) = \vec{h}(\mathbf{k}) \cdot \vec{\gamma}$$

where $h_0(\mathbf{k}) = m_0 - t_0 \sum_j \cos(k_j)$ $\quad h_j(\mathbf{k}) = t_{\text{so}} \sin(k_j)$

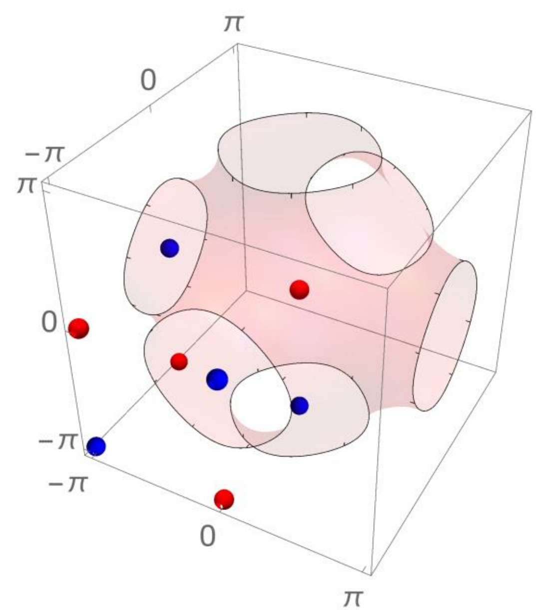
γ matrices
$$\gamma_0 = \sigma_z \otimes \tau_x$$
$$\gamma_1 = \sigma_x \otimes \mathbf{1}$$
$$\gamma_2 = \sigma_y \otimes \mathbf{1}$$
$$\gamma_3 = \sigma_z \otimes \tau_z$$

obey the Clifford algebra $\{\gamma_i, \gamma_j\} = 2\delta_{ij}\mathbf{1}$

a chiral symmetry defined by $S = \sigma_z \otimes \tau_y$

$$S\mathcal{H}(\mathbf{k})S^{-1} = -\mathcal{H}(\mathbf{k})$$

AIII class



- charge -1 $(0,0,0)$ $(-\pi,-\pi,0)$ $(-\pi,0,-\pi)$ $(0,-\pi,-\pi)$
- charge +1 $(-\pi,-\pi,-\pi)$ $(0,0,-\pi)$ $(0,-\pi,0)$ $(-\pi,0,0)$

Numerical results

- 3D model

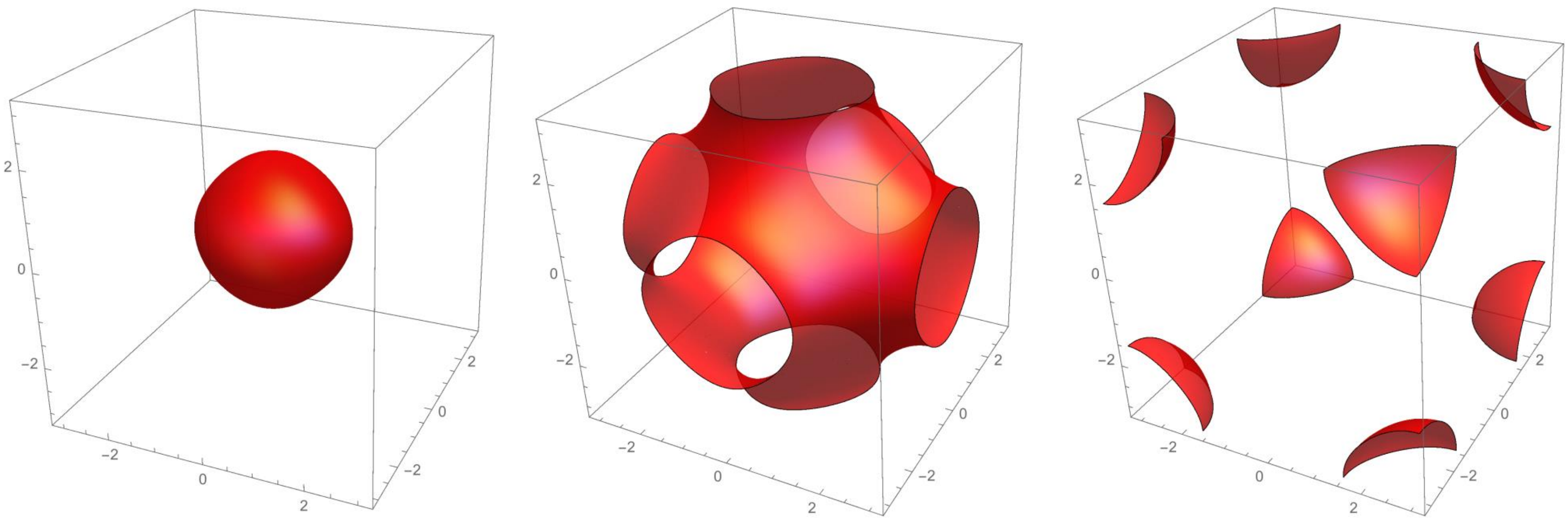
Topological phases:

(I) $t_0 < m_z < 3t_0$ with winding number $v_3 = -1$;

(II) $-t_0 < m_z < t_0$ with $v_3 = 2$;

(III) $-3t_0 < m_z < -t_0$ with $v_3 = -1$

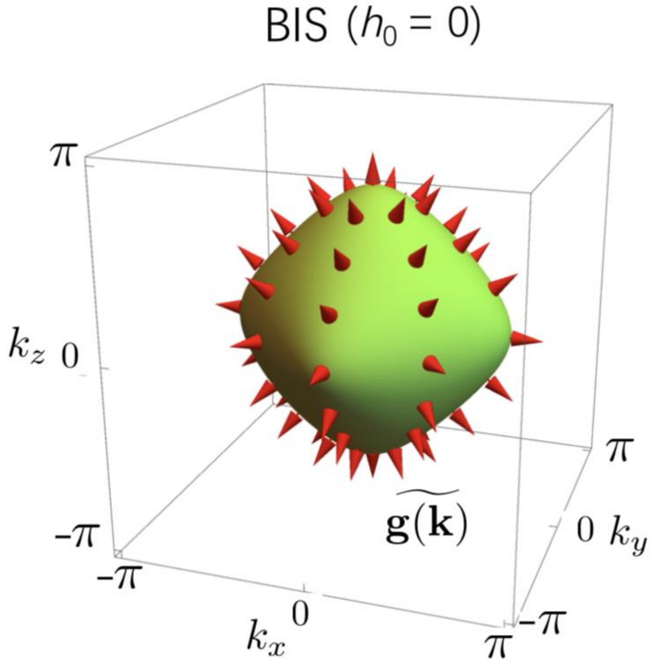
$t_0 < m_z < 3t_0$ $\qquad$ $-t_0 < m_z < t_0$ $\qquad$ $-3t_0 < m_z < -t_0$

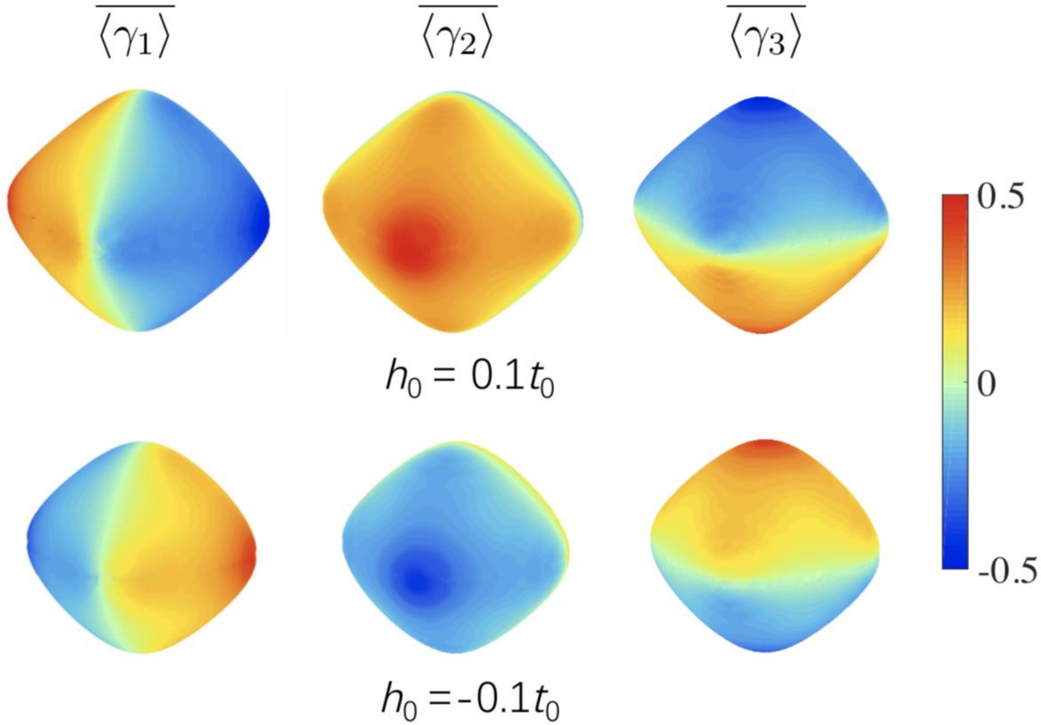


Numerical results

- 3D model

$m_0 = 8t_0 \to 1.3t_0$





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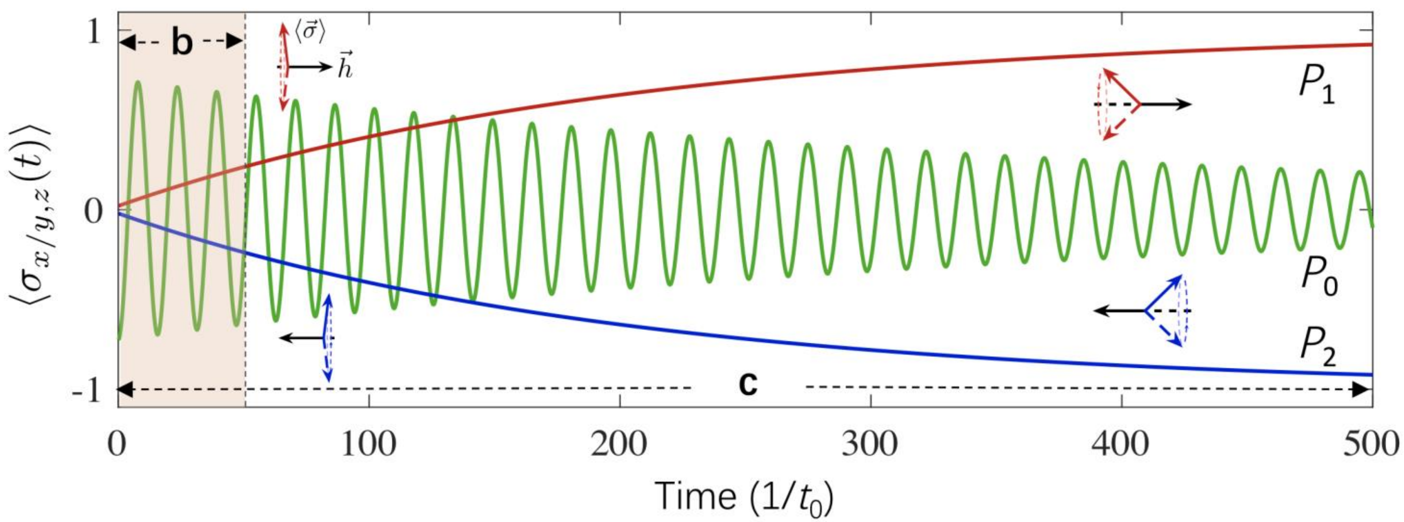
Dissipative dynamics

- Lindblad master equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{\mathbf{k}} = -\mathbb{i}\left[\mathcal{H},\rho_{\mathbf{k}}\right] + \eta\left(\tilde{\sigma}_{-}\rho_{\mathbf{k}}\tilde{\sigma}_{+} - \frac{1}{2}\left\{\tilde{\sigma}_{+}\tilde{\sigma}_{-},\rho_{\mathbf{k}}\right\}\right)$$

$$\rho_{\mathbf{k}}(0) = f(E_{+},T)\left|+,\mathbf{k}\right\rangle\left\langle+,\mathbf{k}\right| + f(E_{-},T)\left|-,\mathbf{k}\right\rangle\left\langle-,\mathbf{k}\right|$$

$\tilde{\sigma}_{\pm} \equiv (\tilde{\sigma}_x \pm \mathbb{i}\tilde{\sigma}_y)/2$ in the eigenbasis of post-quench Hamiltonian



Dissipative dynamics

- Spin textures

Short time averages determine the BIS by $\overline{\langle \vec{\sigma} \rangle} \simeq 0$

Long time averages reflect the vector field $\vec{h}(\mathbf{k}) = \mathbf{h}_{\rm so}(\mathbf{k})$ by

$$h_{x,y} \sim -\overline{\langle \sigma_{x,y} \rangle}$$