Dipolar bosons: from solitons to rotons

Kazimierz Rzążewski
Center for Theoretical Physics
Polish Academy of Sciences
Warsaw, Poland

NARODOWE CENTRUM NAUKI

Vilnius, July 30, 2018

in search of stronger dipolar gases…

Chromium: 2005, Tilman Pfau, Stuttgart

$$\mu_{Cr} = 6\mu_B$$

Erbium, 2012, Francesca Ferlaino, Innsbruck

$$\mu_{Er} = 7\mu_B$$

Dysprosium, 2011, Ben Lev, Urbana-Champaign

$$\mu_{Dy} = 10\mu_B$$

dark solitons-contact interactions

Hartree wave function for N bosons:

$$\Phi(x_1, x_2, \ldots, x_N, t) = \prod_{i=1}^{N} \varphi(x_i, t)$$

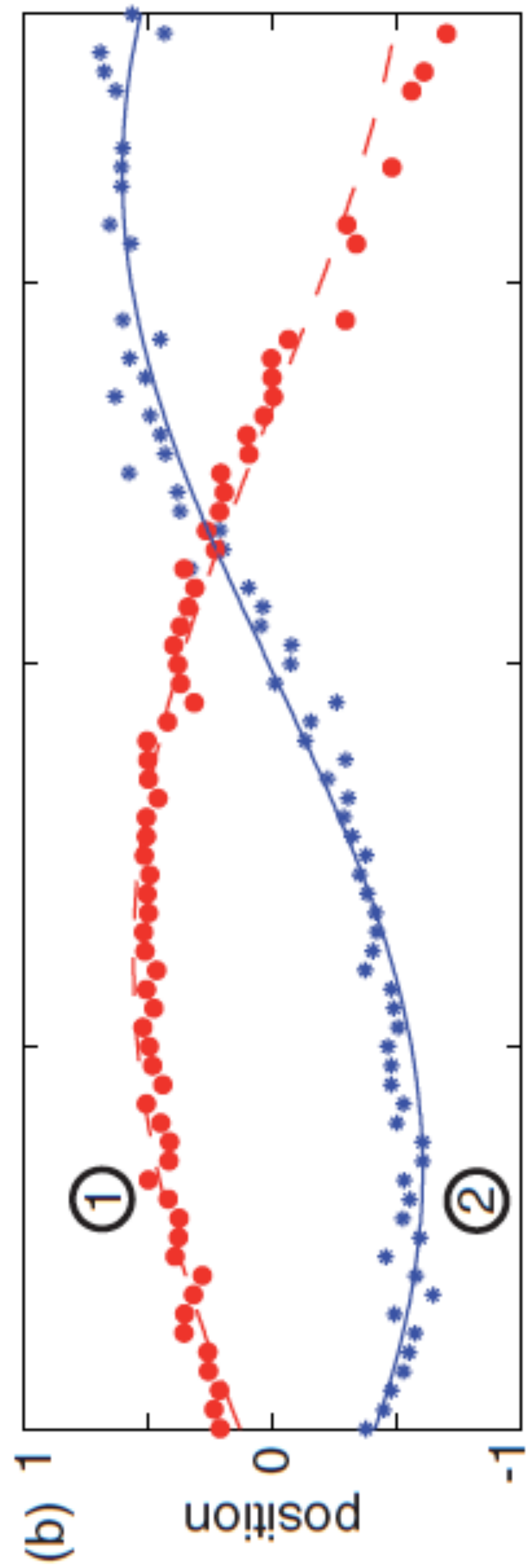
BEC equation - Gross-Pitaevski equation

$$i\hbar\dot{\varphi}(x,t) = \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + g|\varphi|^2 \right]\varphi(x,t)$$

on a line is of soliton category, but....

- g>0 - dark solitons
- never infinite line
- always 3D
- (almost) always trap (additional harmonic potential)

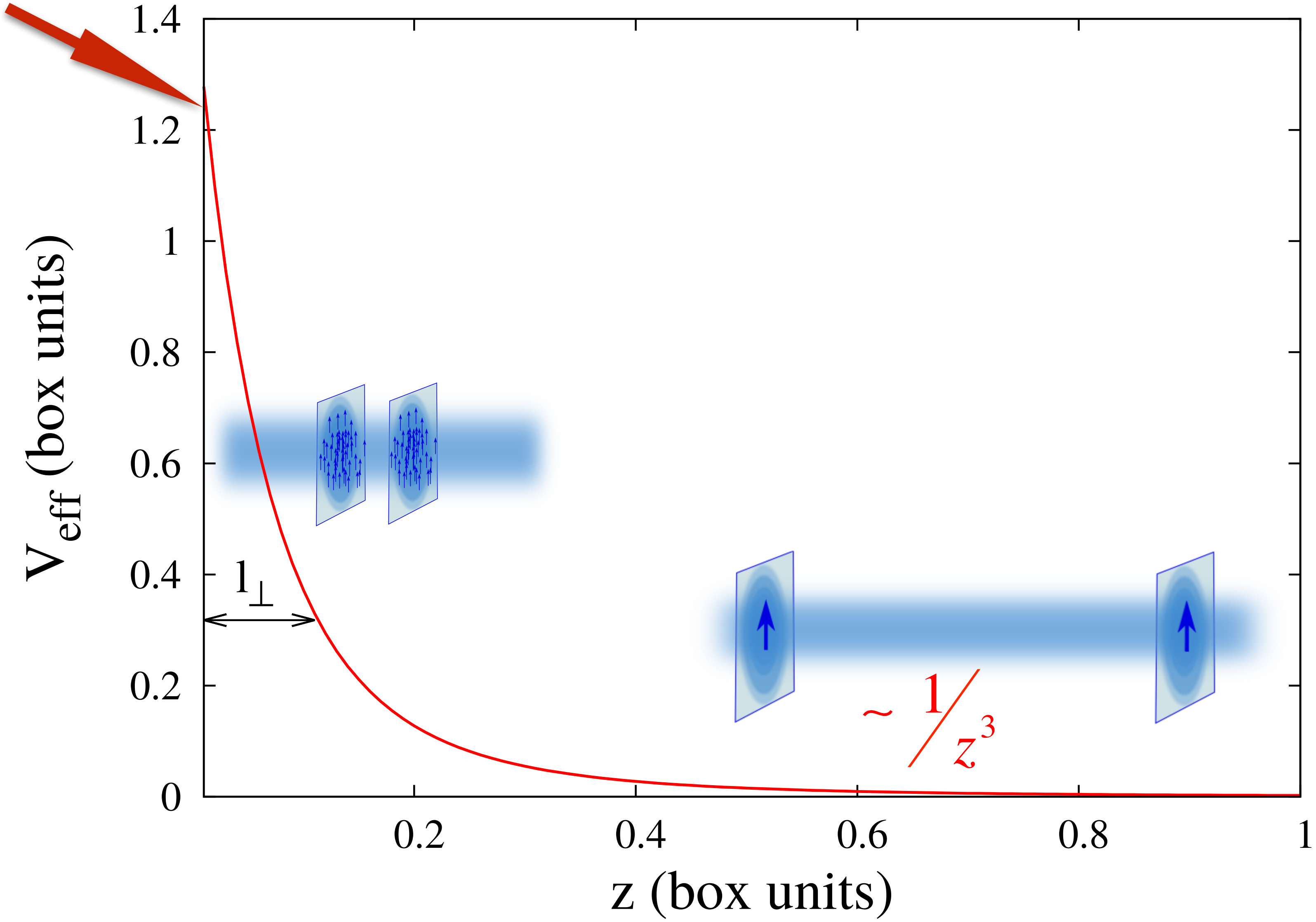
collision of dark solitons



S. Stellmer, C. Becker, P. Soltan-Panahi, E-M. Richter, S. Dörscher, M. Baumert, J. Kronjäger, K. Bongs, and K. Sengstock, Collisions of Dark Solitons in Elongated Bose-Einstein Condensates. PRL, **101**, 120406 (2008)

effective 1D potential

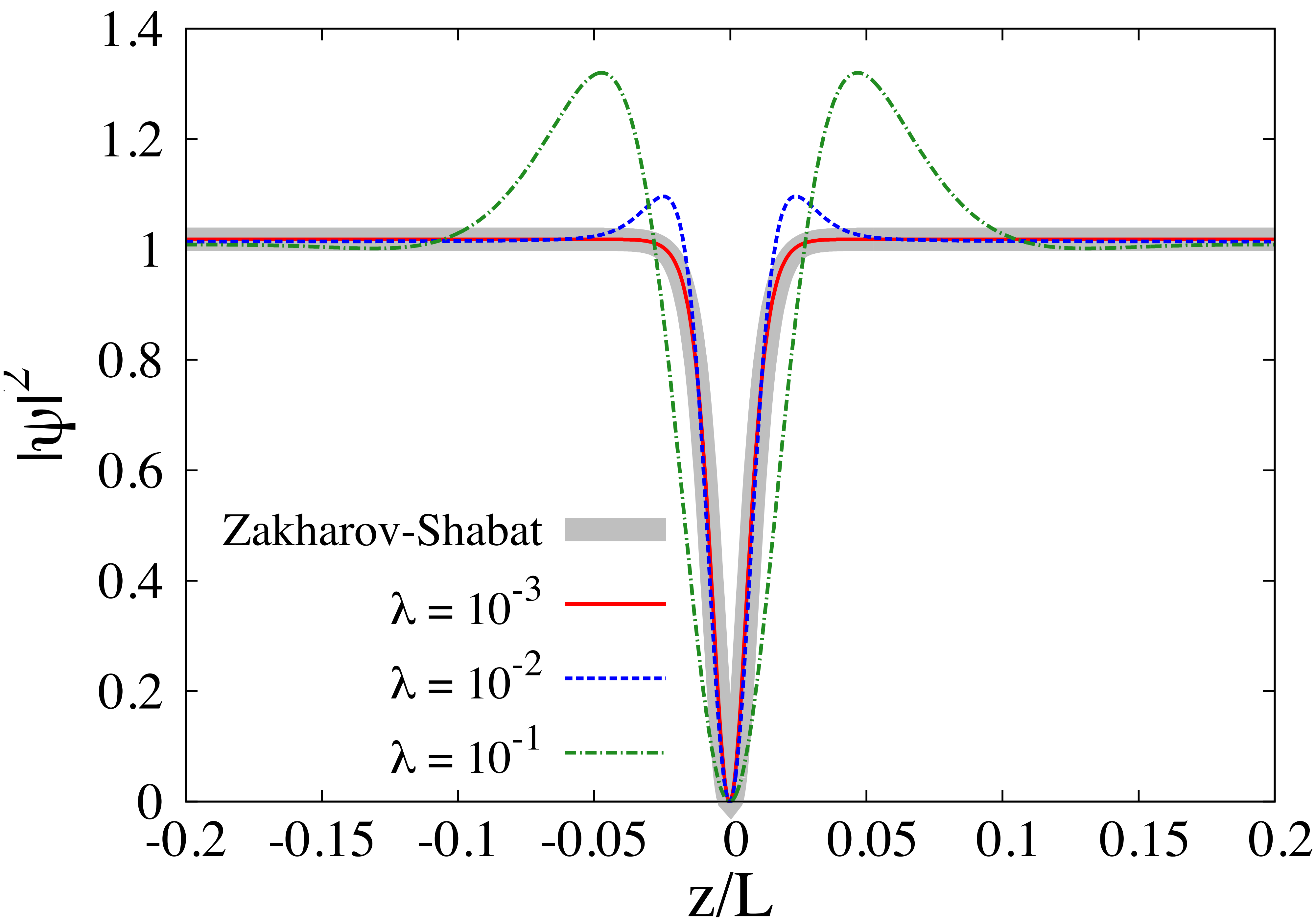
finite value



(kill the contact term by Feshbach resonance)

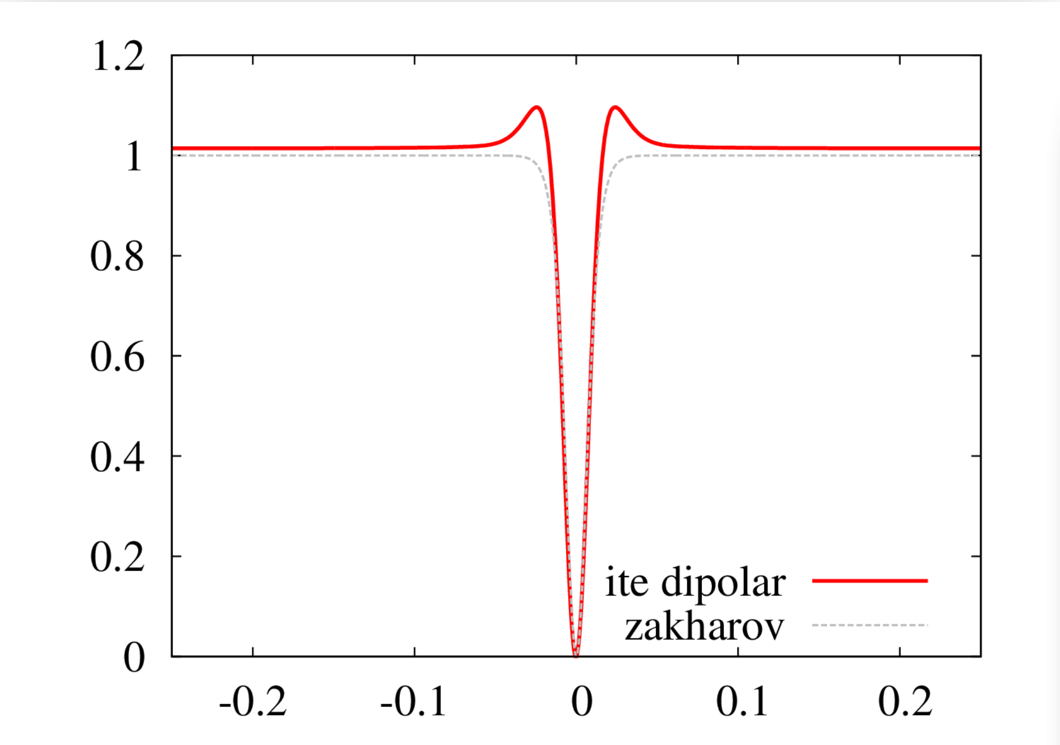
single dipolar soliton
1D - box with periodic boundary conditions

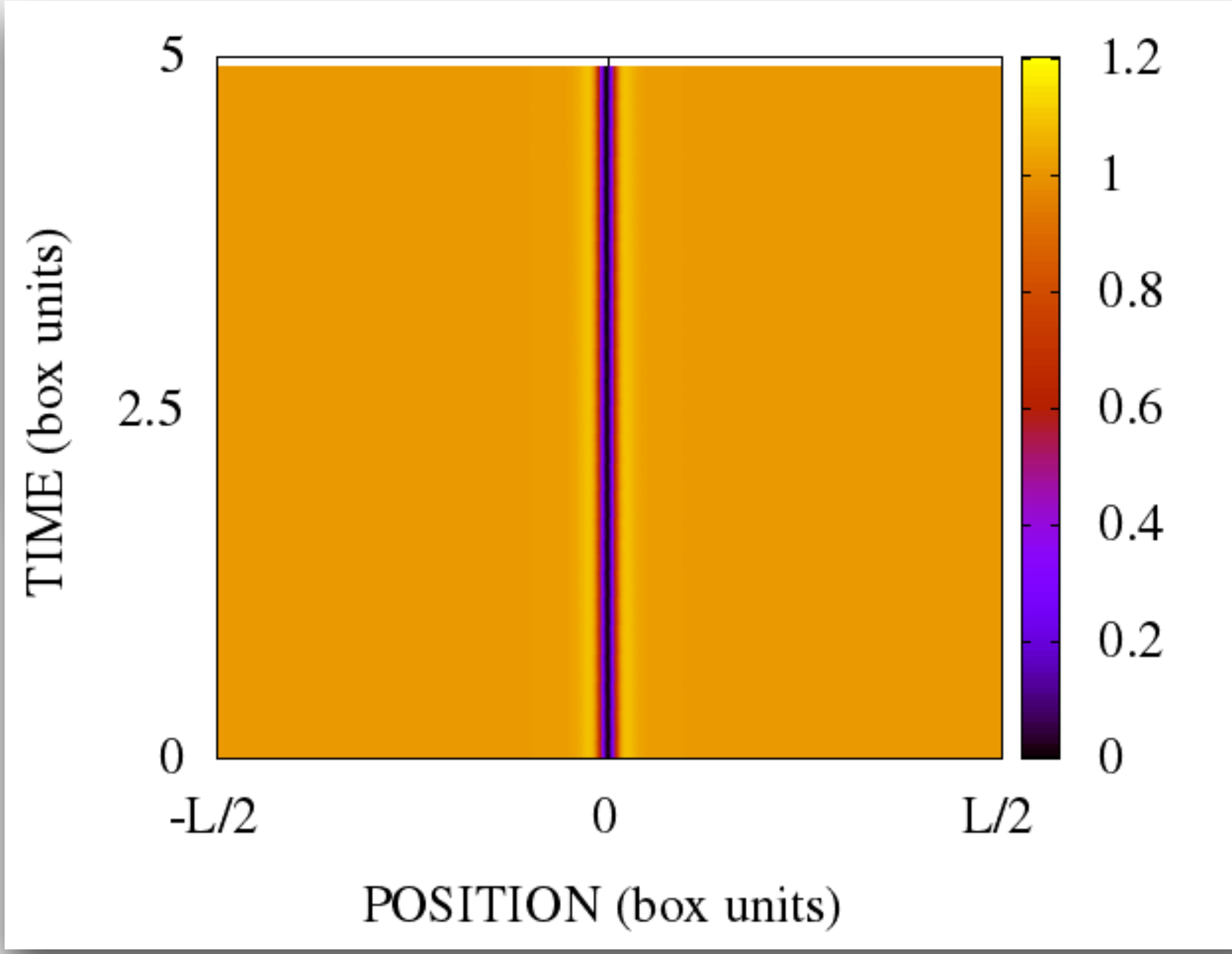
constraint: $\varphi(z)=\frac{\pi}{2}\left(\frac{2z}{L}-\mathrm{sgn}(z)\right)$



evolution of a single dipolar soliton

(in a co-moving frame)

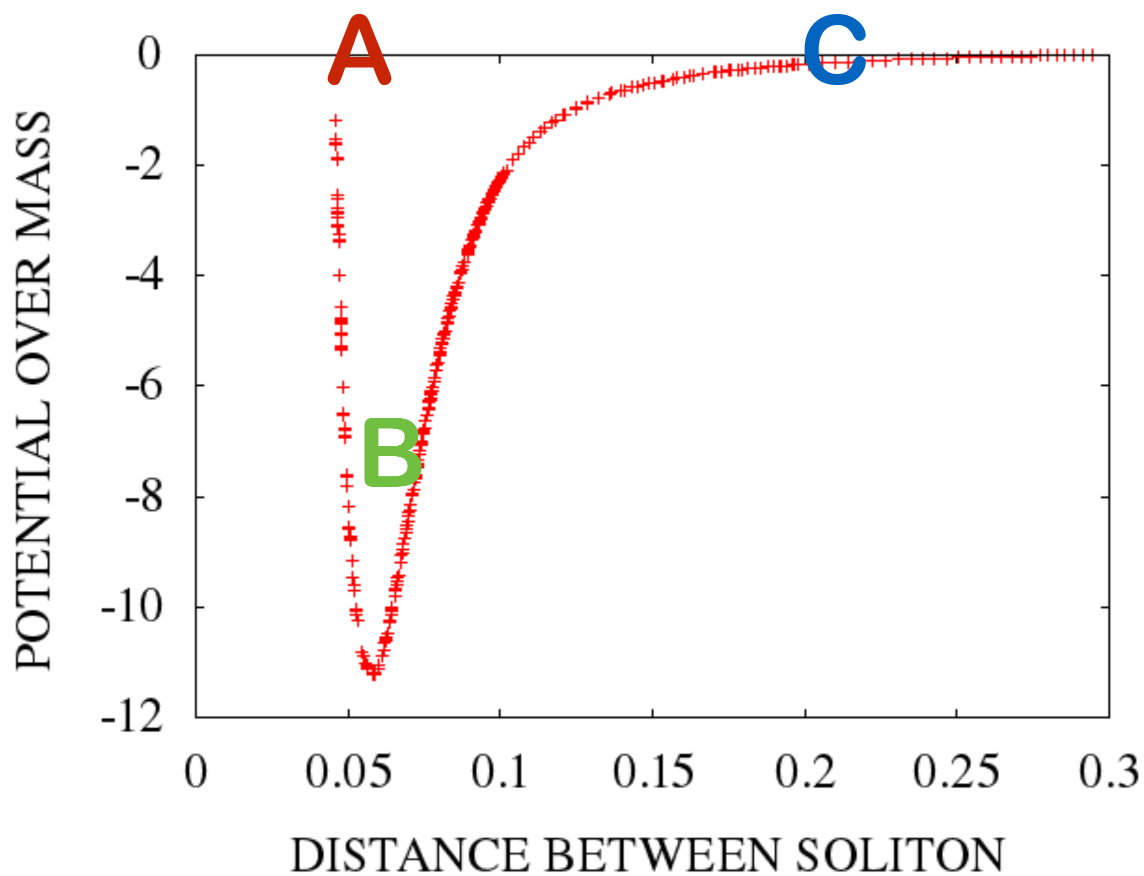




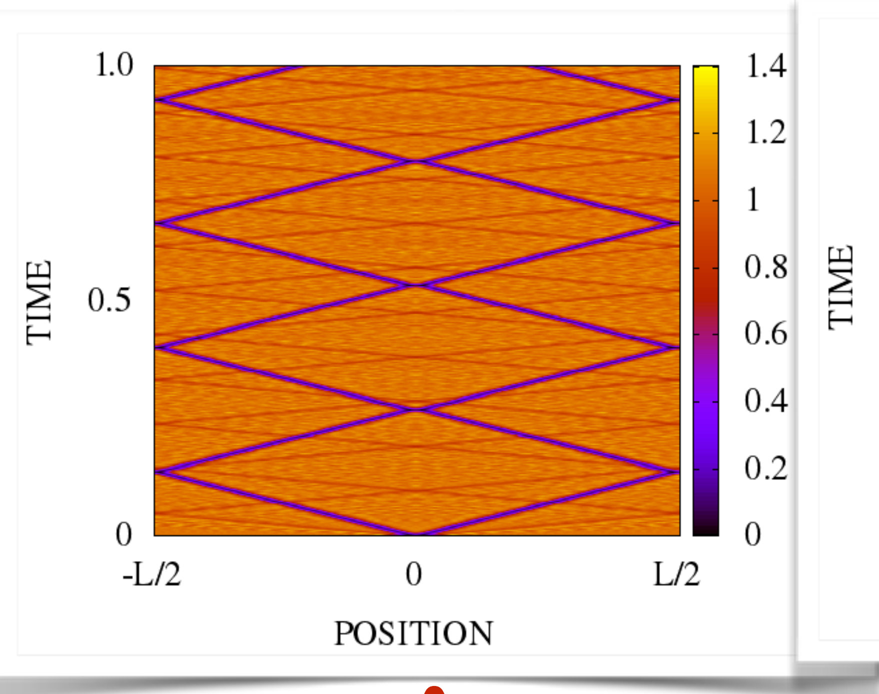
soliton width

$$\xi^2 = \frac{1}{\sqrt{3\frac{N}{L}a_{\mathrm{dd}}}} l$$

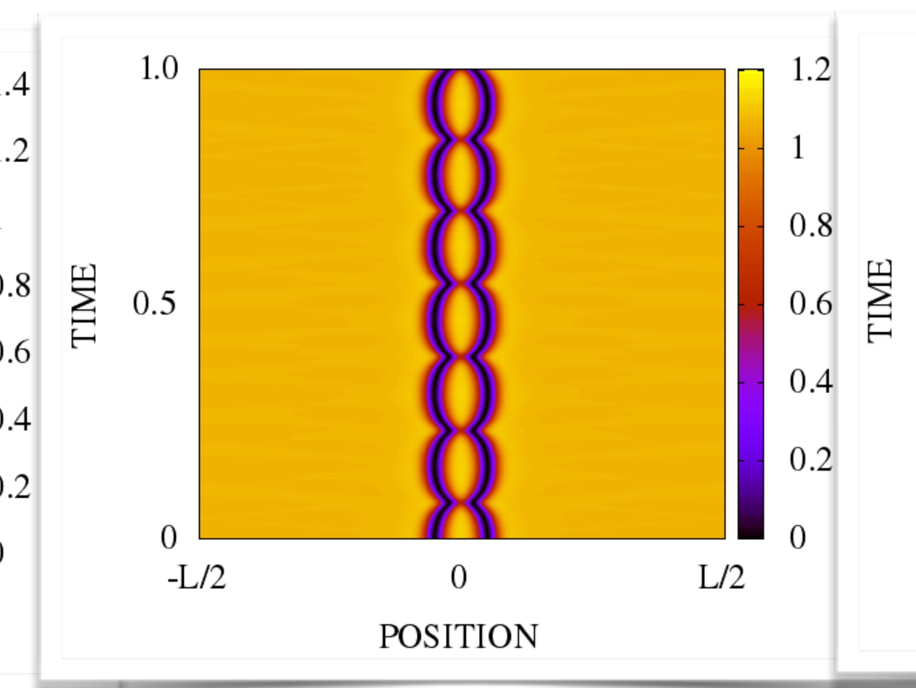
inter-soliton potential



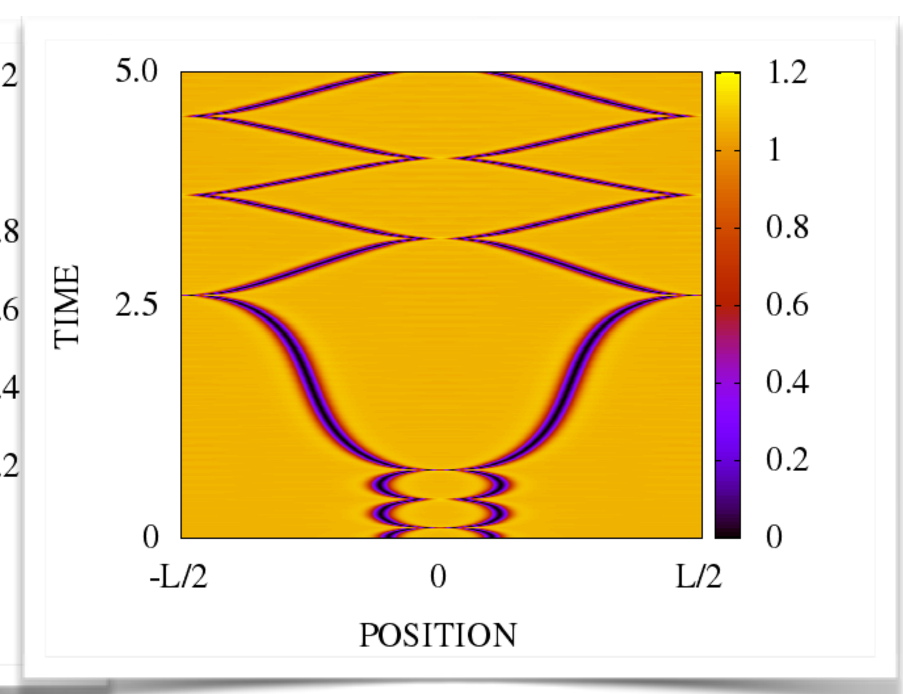
collisions elastic?



A

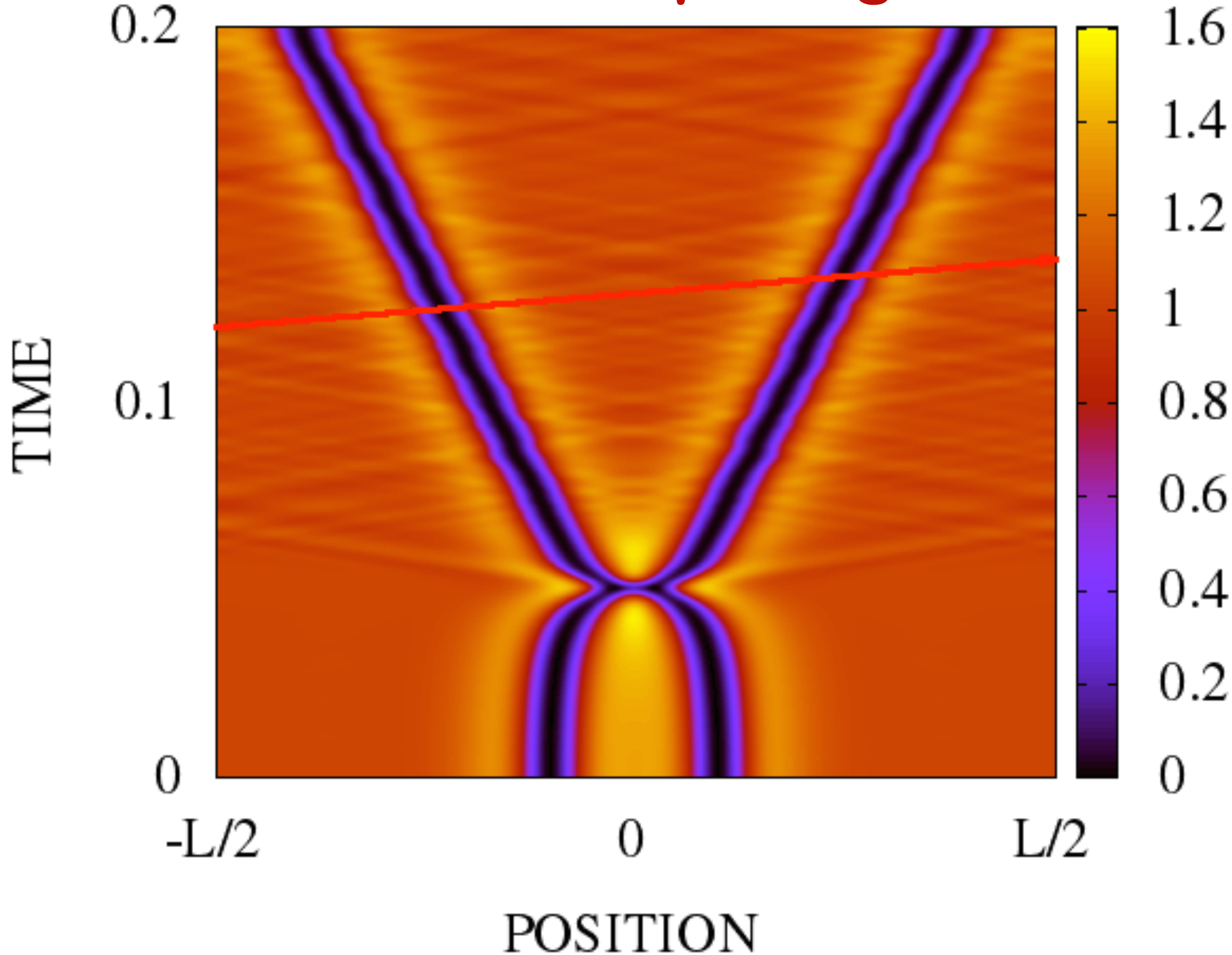


B



C

collision of two solitons in a 1D dipolar gas



K. Pawłowski and K. Rz. New J. Phys. 17 (2015) 105006)

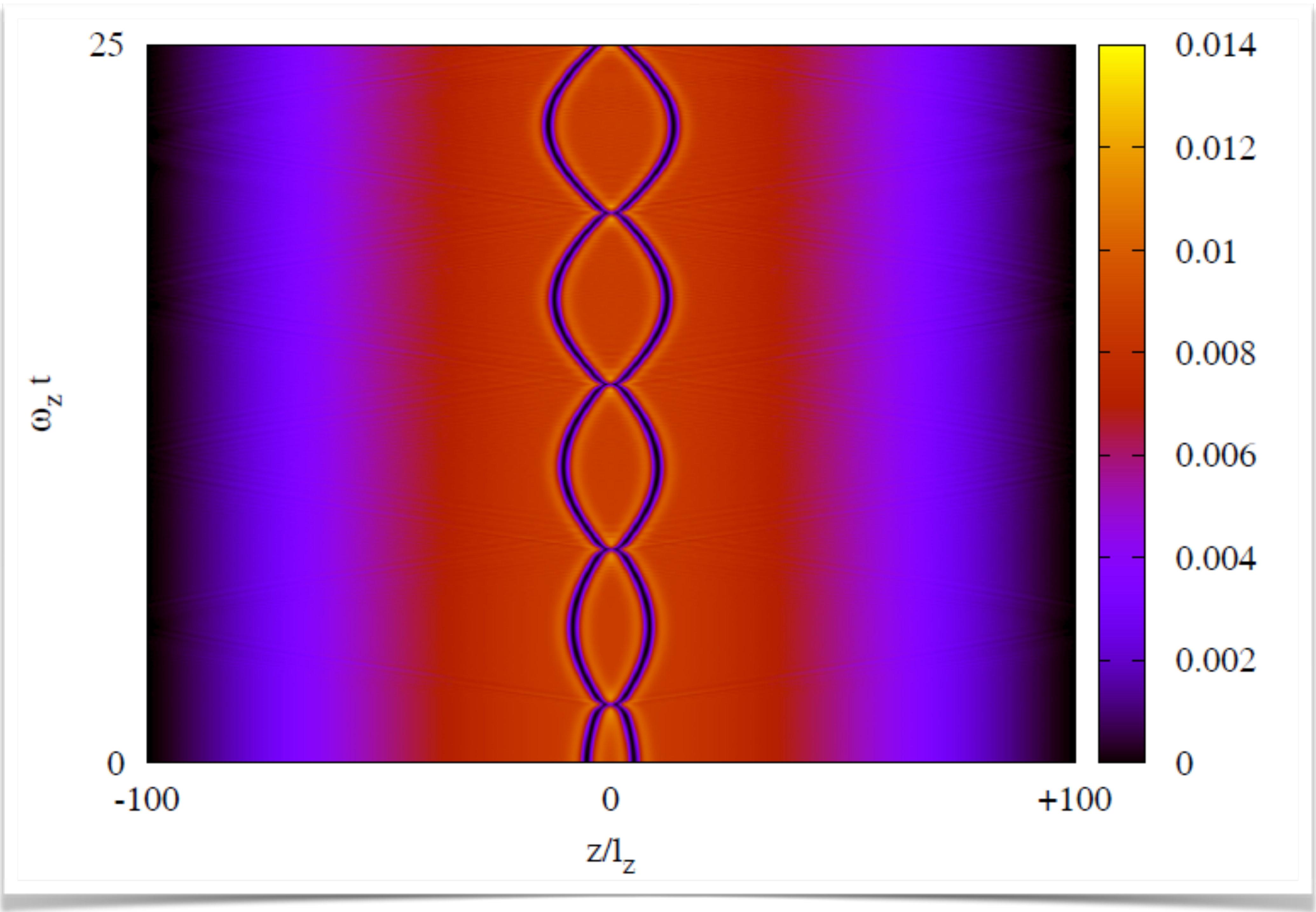
How about realistic trapping potential?

N=10000 Dysprosium atoms

$$(\omega_x, \omega_y, \omega_z) = 2\pi(128, 128, 2)Hz$$

T. Bland, K. Pawłowski, M. J. Edmonds, K. Rzążewski, and N. G. Parker, Anomalous oscillations of dark solitons in trapped dipolar condensates, Phys. Rev. A, 95, 063622 (2017)

inelastic collisions of dipolar solitons in a 3D trap



$$\varepsilon_{dd} = \frac{a_{dd}}{a_s} \quad a_{dd} = \frac{m\mu_0\mu^2}{12\pi\hbar^2}$$

$\omega_z = 2\pi 2\mathrm{Hz}$

$\omega_x = \omega_y = 2\pi 128\mathrm{Hz}$

N=10 000

3D

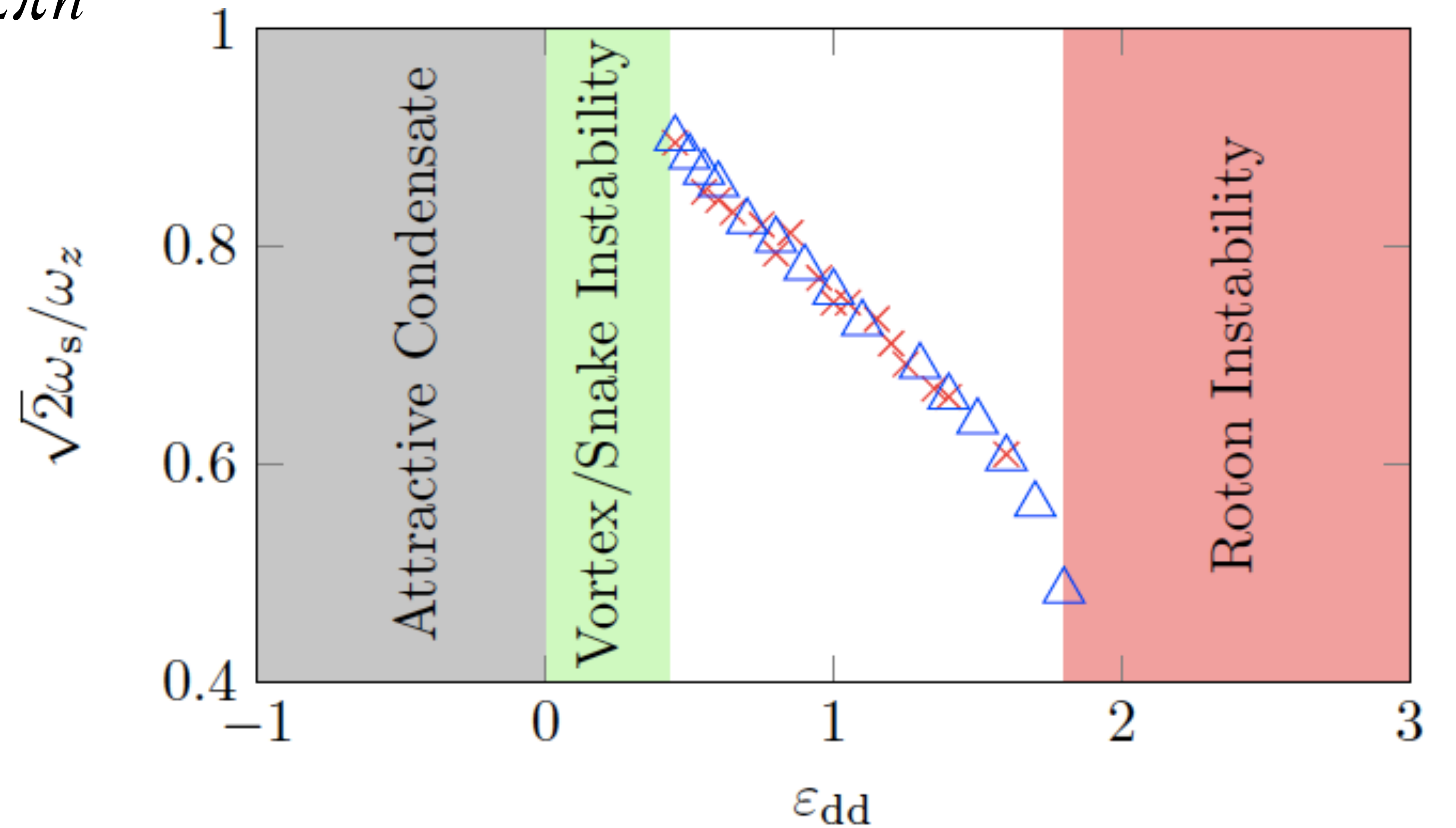


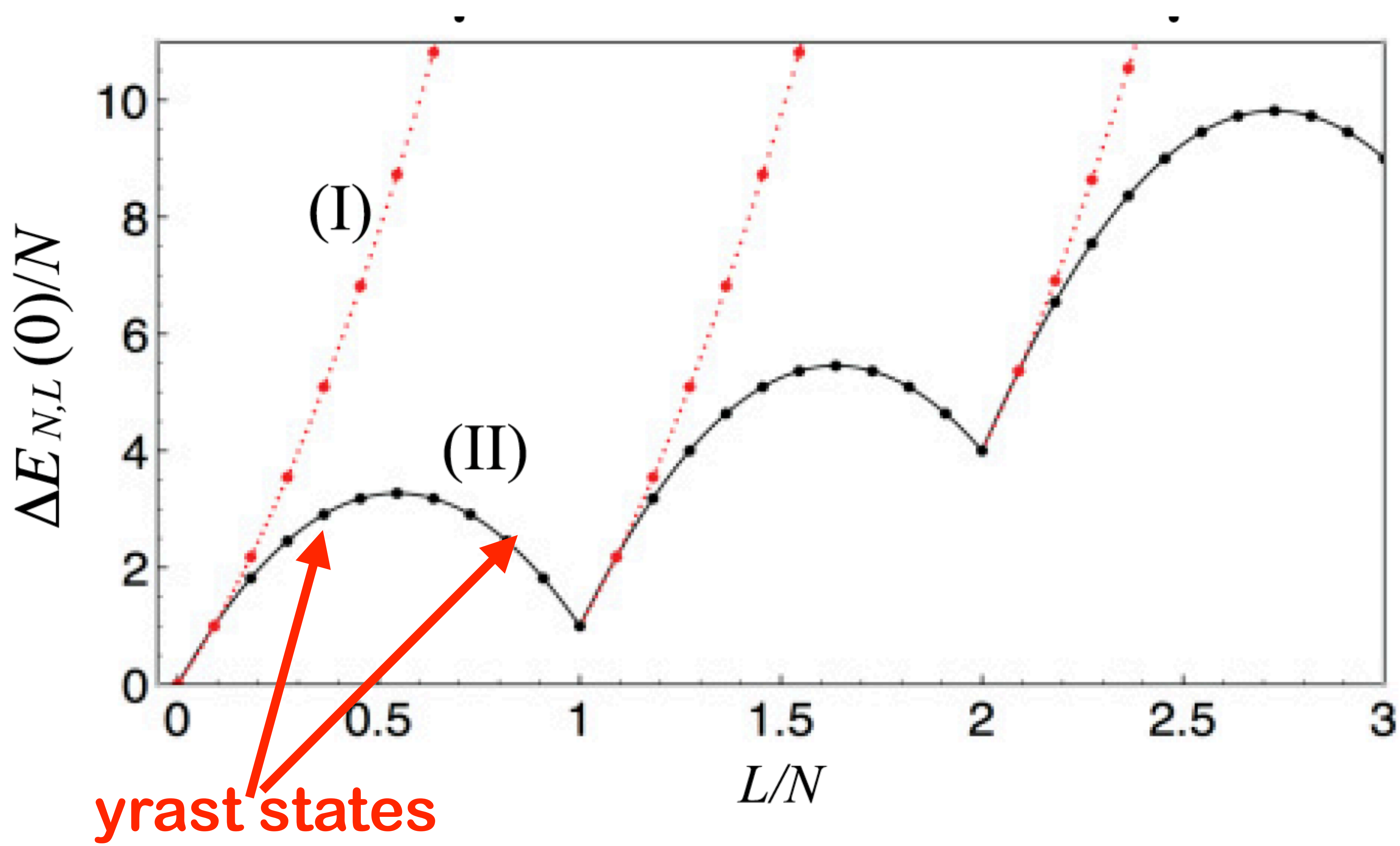
FIG. 4. (Color online) Oscillation frequency and phase d gram for a dark soliton in a [164]Dy BEC with Feshbach tuni of a_s, based on a recent experiment set-up [43]. Shown a cases where the soliton is imposed in the initial conditi (blue triangles, as per Fig. 1) and imprinted in real ti (red crosses). Outside of the dark soliton regime, the co

Lieb-Liniger model (1963)

- 1D model, no trap
- *N bosonic* atoms
- periodic boundary condition
- contact, repulsive, interaction potential

$$\left[-\sum_{j=1}^{N}\frac{\partial^2}{\partial x_j^2}+2c\sum_{j>l}\delta(x_j-x_l)\right]\Psi=E\Psi$$

small number of atoms in a ring trap

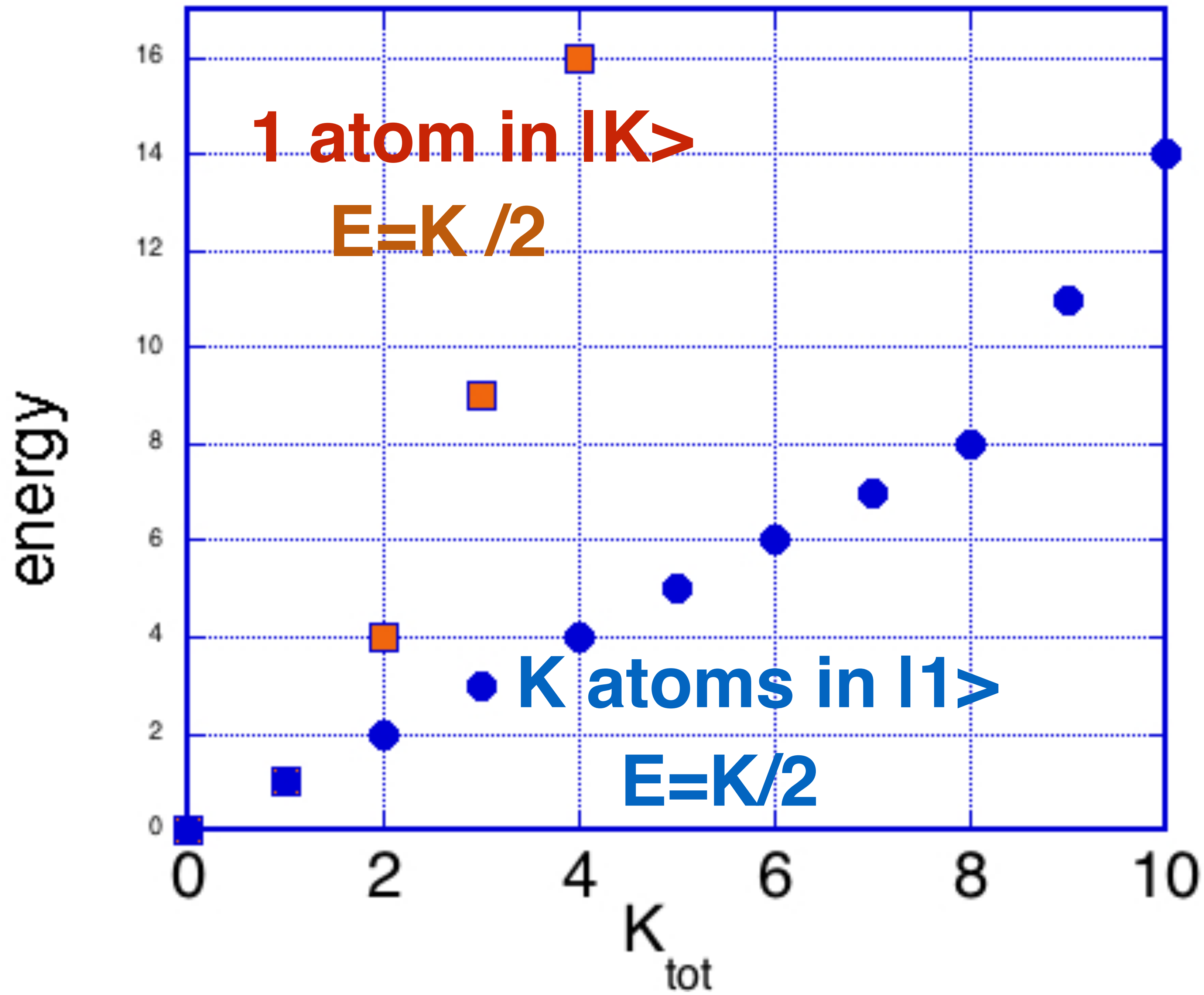


R. Kanamoto, L. D. Carr, and M. Ueda, *Phys. Rev. A*, **81**, 023625 (2010).

secret of two types of excitations revealed

ideal gas

N=8

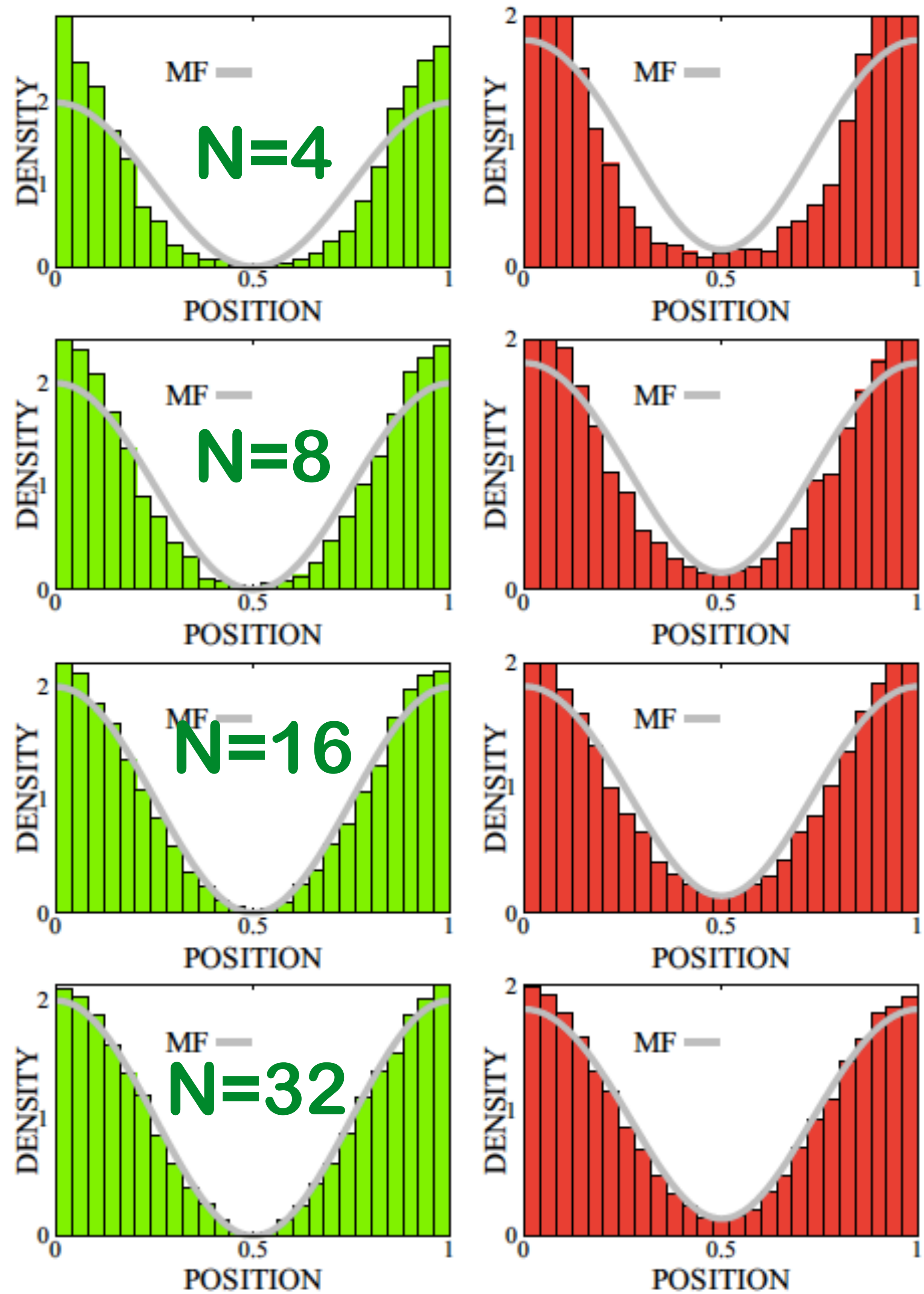


wave function of a “dark soliton”

$$= \frac{1}{\sqrt{L^N \binom{N}{N/2}}} \sum_{\sigma} e^{i2\pi(x_{\sigma(1)}+x_{\sigma(2)}+\ldots+x_{\sigma(N/2)})/L},$$

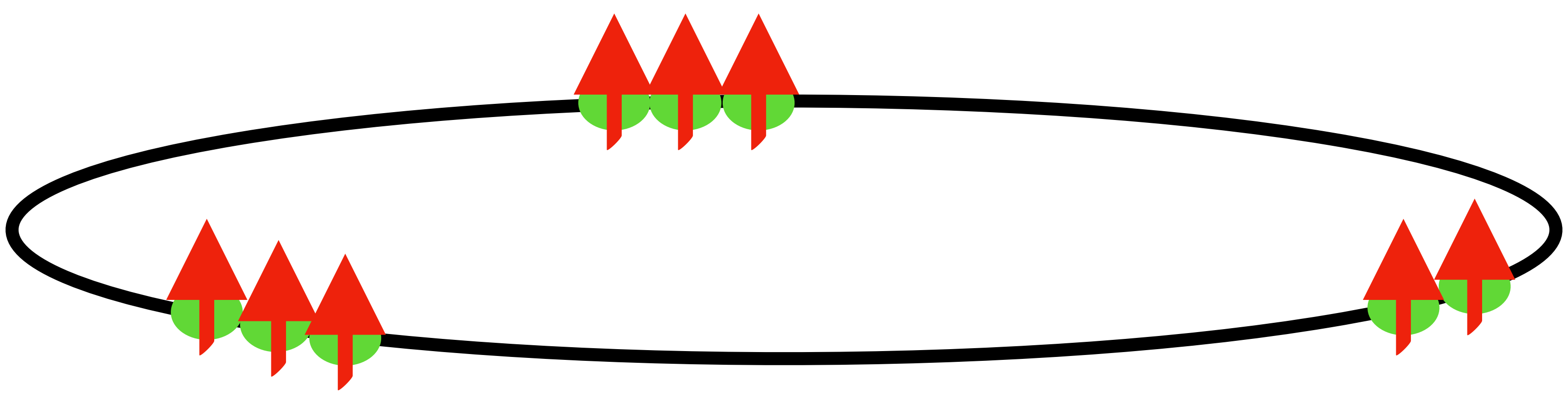
wave function of the last atom

$$\psi_{\text{con}}^{x_1,\ldots,x_{N-1}}(x_N) \propto 1 + e^{2i\pi(x_N+X)/L},$$



R. Ołdziejewski, W. Górecki, K. Pawłowski, and K. Rzążewski, Many-body solitonlike states of the bosonic ideal gas *Phys. Rev. A* , **97**, 063617 (2018)

roton



Roton in a many-body dipolar system

Rafał Ołdziejewski, Wojciech Górecki, Krzysztof Pawłowski, K. Rz.

arXiv:1801.06586

Hamiltonian first!

$$\hat{H} = \sum_k \frac{\hbar^2 k^2}{2m} \hat{a}_k^\dagger \hat{a}_k + \frac{1}{2} \sum_{k_1,k_2,k} \hat{a}_{k_1+k}^\dagger \hat{a}_{k_2-k}^\dagger V_{\text{eff}}(k) \hat{a}_{k_1} \hat{a}_{k_2}, \tag{1}$$

Bogoliubov spectrum

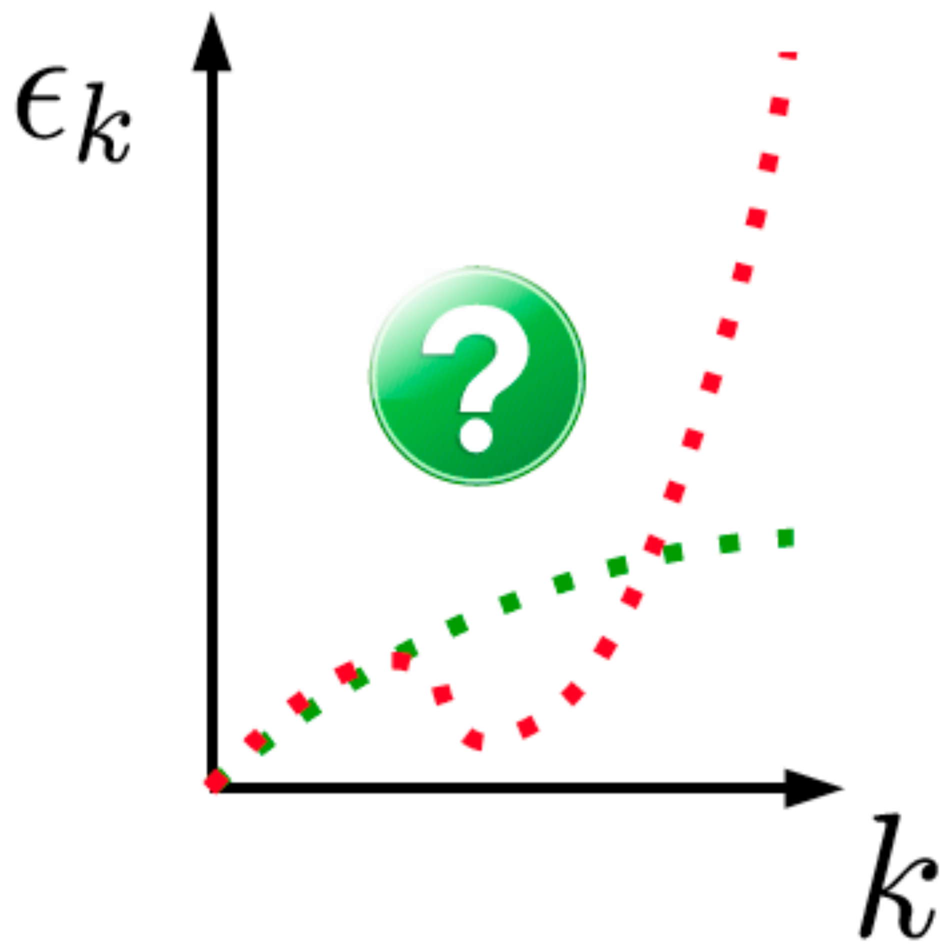
$$\epsilon_k = \sqrt{\frac{k^2}{2}\left(\frac{k^2}{2} + 2NV_{\text{eff}}(k)\right)}$$

L. Santos, G. V. Shlyapnikov, and M. Lewenstein, *PRL* **90**, 250403 (2003)

number conserving Bogoliubov vacuum:

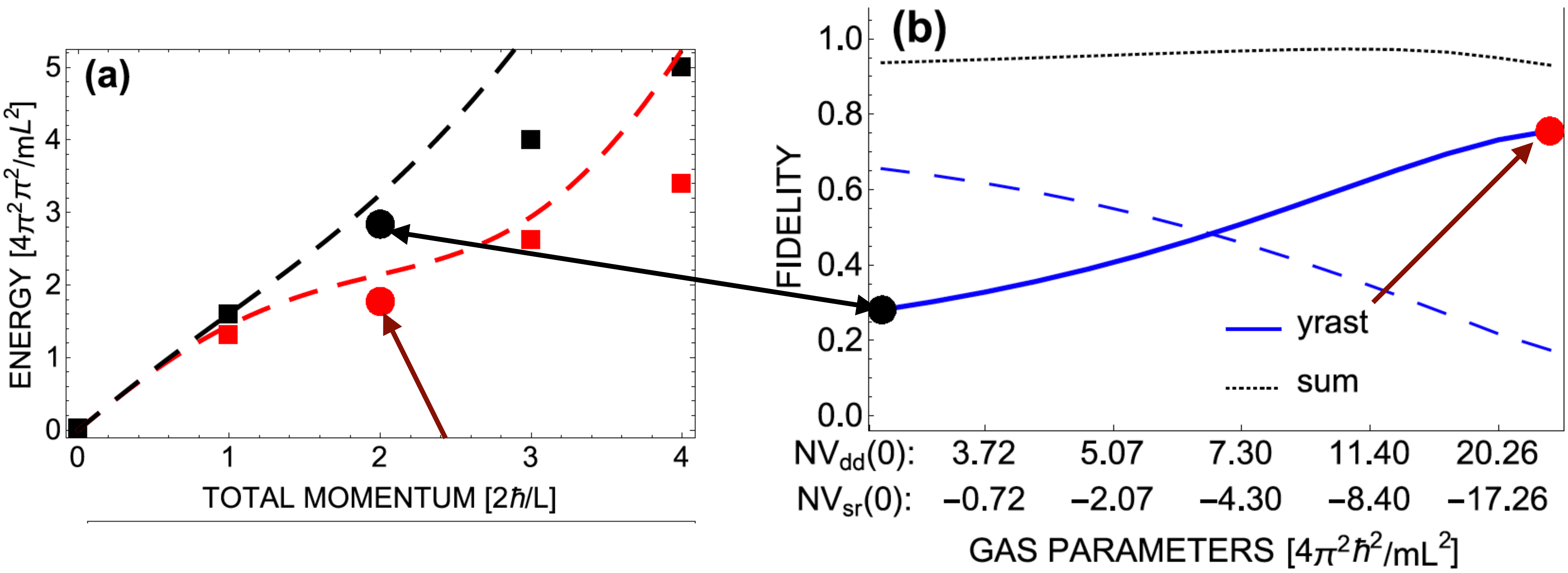
$$|0\rangle_B \propto \left(\left(\hat{a}_0^\dagger\right)^2 + 2\sum_{k>0}^{\infty} \frac{v_k}{u_k} \hat{a}_k^\dagger \hat{a}_{-k}^\dagger\right)^{N/2} |\text{vac}\rangle$$

Can roton appear as the yrast state?



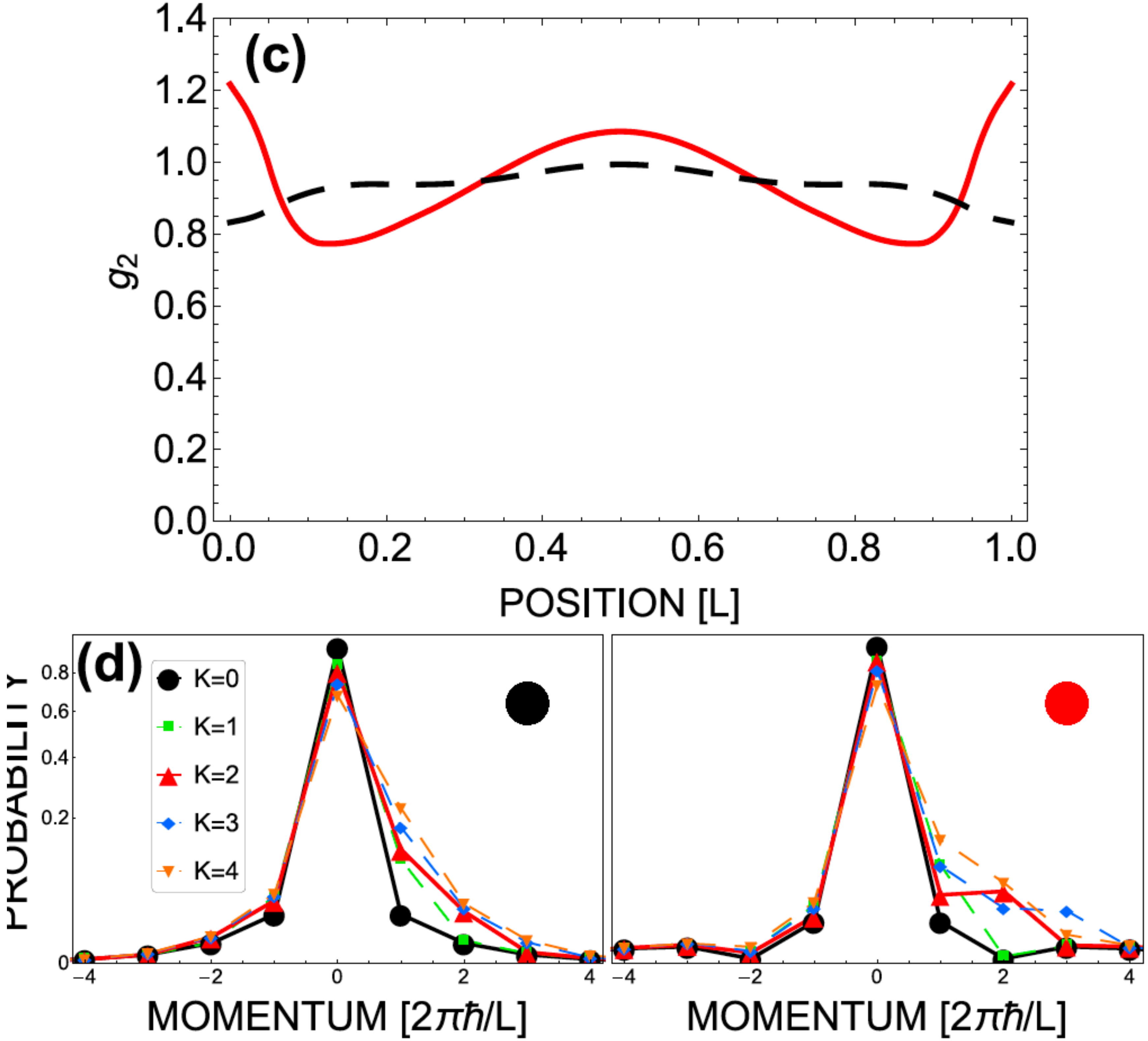
Dysprosium parameters

N=8

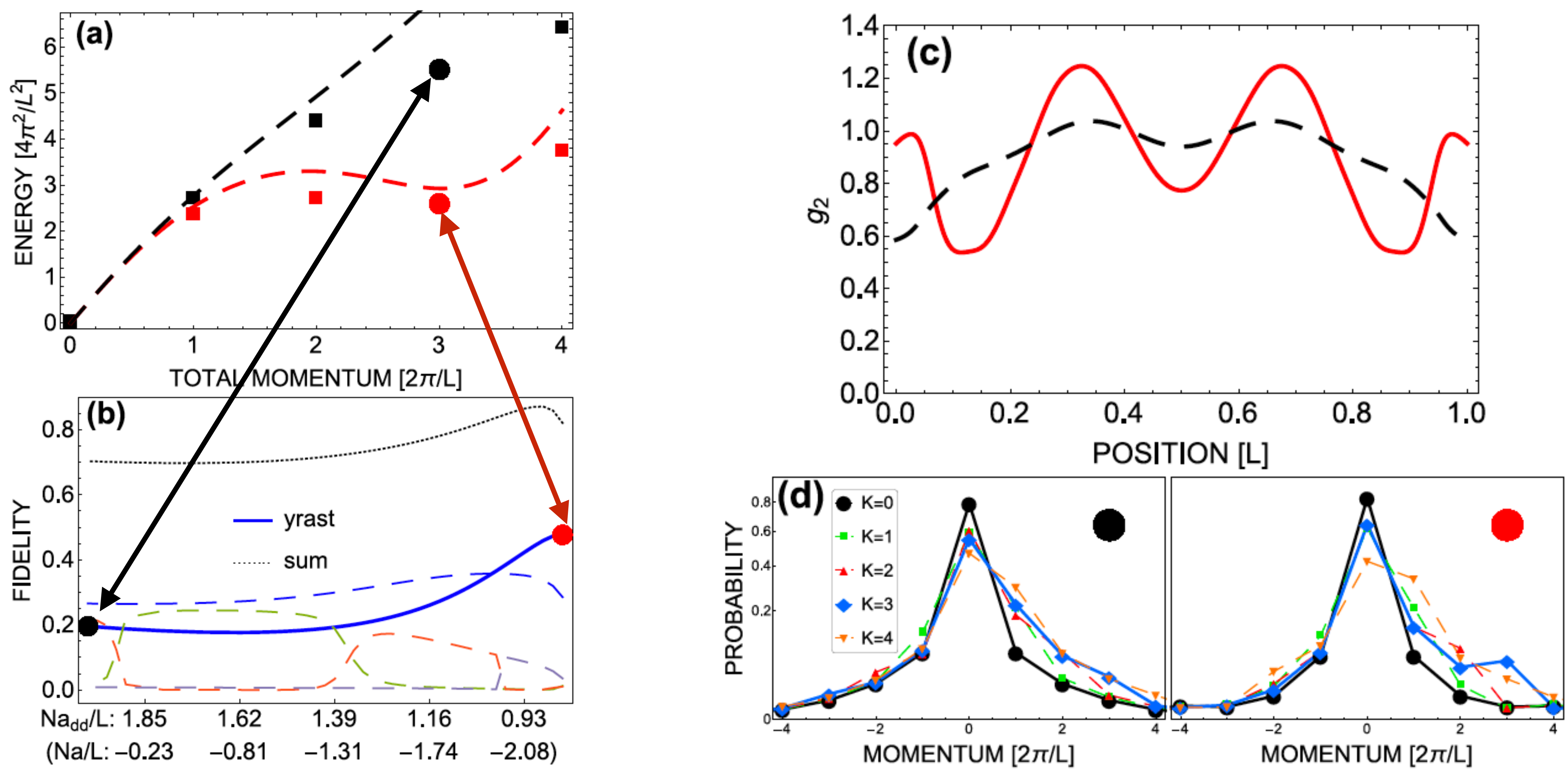


Dysprosium parameters

N=8



deep roton
N=8



conclusions:

- solitons in dipolar gas interact
- their collisions are inelastic
- also in this case dark solitons exist in thermal equilibrium
- their oscillation frequency strongly depends on the strength of dipolar interactions
- few dipolar atoms - a soluble problem with rich structure