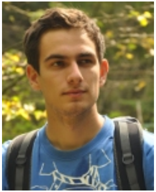
Time Crystal Platform

Krzysztof Sacha

Jagiellonian University in Kraków.

People

Krzysztof Giergiel



Artur Miroszewski

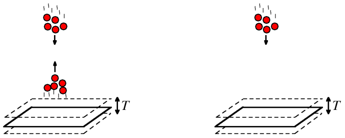
Topological time crystal part:

A. Dauphin

M. Lewenstein

J. Zakrzewski

Discrete time crystals



$|\psi\rangle \propto |N,0\rangle + |0,N\rangle$ $\qquad$ $|\psi\rangle \propto |N,0\rangle$

Theoretical prediction:

K. Sacha, PRA **91**, 033617 (2015).

V. Khemani *et al.*, PRL **116**, 250401 (2016).

D. V. Else *et al.*, PRL **117**, 090402 (2016).

Peter Hannaford,
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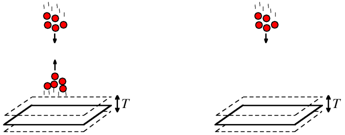
Melbourne

First experiments:

J. Zhang *et al.*, Nature **543**, 217 (2017).

S. Choi *et al.*, Nature **543**, 221 (2017).

Discrete time crystals



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K. Giergiel, A. Kuroś, KS, "*Discrete Time Quasi-Crystals*", arXiv:1807.02105.

Condensed matter physics in time crystals

Platform for time crystal research

Single particle systems

Integrable 1D system:

$$H_0(x,p) \longrightarrow H_0(I) \implies I = const, \quad \theta = \Omega(I)\, t + \theta_0.$$

Time periodic perturbation:

$$H_1 = f(t)\, h(x) \longrightarrow H_1 = \left(\sum_k f_k e^{ik\omega t}\right)\left(\sum_n h_n e^{in\theta}\right).$$

Platform for time crystal research

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Assume s:1 resonance, $\omega = s\,\Omega(I)$. In the moving frame $\Theta = \theta - \frac{\omega}{s} t$

$$H \approx \frac{P^2}{2m_{eff}} + \sum_k f_{-k}\, h_{ks}\, e^{iks\Theta}.$$

Platform for time crystal research

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For example for $f(t) = \lambda \cos(\omega t)$, we get $H \approx \frac{P^2}{2m_{eff}} + V_0 \cos(s\Theta)$.

Crystalline structure in time

A particle bouncing on an oscillating mirror

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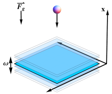

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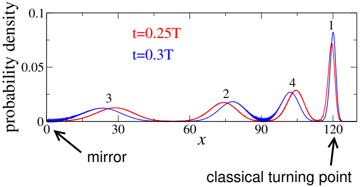

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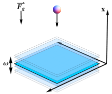

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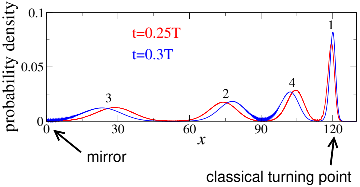

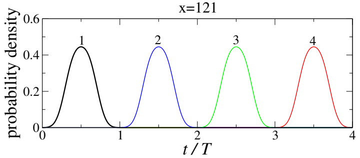

$$E_F = \int_0^{sT} dt \langle \psi | H_F | \psi \rangle \approx -\frac{J}{2} \sum_{j=1}^{s} (a_{j+1}^* a_j + \text{c.c.})$$

$$J = -2 \int_0^{sT} dt \langle \phi_{j+1} | H_F | \phi_j \rangle$$

KS, Sci. Rep. **5**, 10787 (2015).

Topological time crystals

A particle bouncing on an oscillating mirror

Mirror oscillations $\propto \lambda \cos(s\omega t) + \lambda_1 \cos(s\omega t/2)$

$$\textbf{SSH model:} \quad H \approx -\sum_{i=1}^{s/2} \left(J\, b_i^* a_i + J'\, a_{i+1}^* b_i \right)$$

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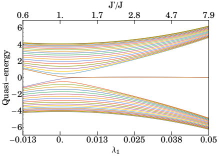
$f(t)$ creates the edge in time:



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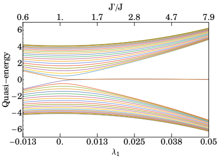

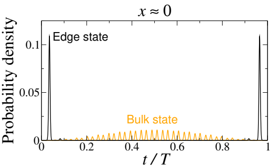

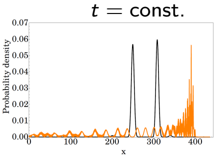

K. Giergiel, A. Dauphin, M. Lewenstein, J. Zakrzewski, KS, arXiv:1806.10536

Quasi-crystals in the time domain

A particle bouncing on an oscillating mirror

Fibonacci quasi-crystal (the inflation rule $B \to BS$ and $S \to B$):

$B \to BS \to BSB \to BSBBS \to BSBBSBSB \to \dots$

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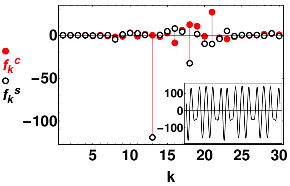

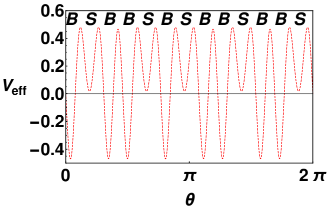

K. Giergiel, A. Miroszewski, KS, PRL **120**, 140401 (2018).

Exotic Interactions

Ultra-cold atoms bouncing on an oscillating mirror

Bosons:

$$\hat{H}_F = -\frac{J}{2}\sum_{j=1}^{s}(\hat{a}^\dagger_{j+1}\hat{a}_j + \mathrm{h.c.}) + \frac{1}{2}\sum_{i,j=1}^{s} U_{ij}\ \hat{a}^\dagger_i\hat{a}^\dagger_j\ \hat{a}_j\hat{a}_i$$

$$U_{ij} \propto \int_0^{sT} dt\ g_0\ \int dx\ |\phi_i|^2\ |\phi_j|^2,$$

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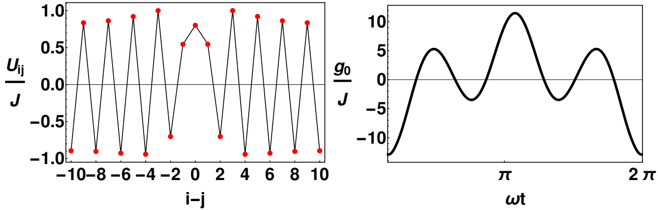
20:1 resonance



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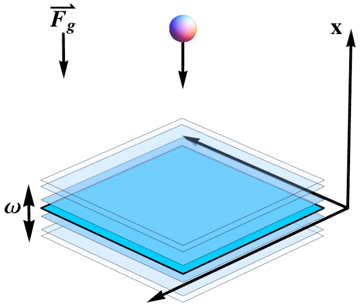
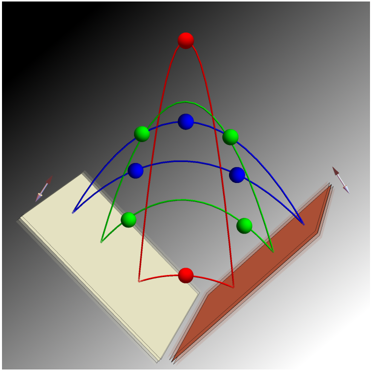
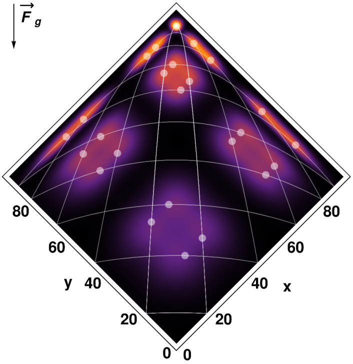
Time crystals with properties of 2D space crystals





Time crystals with properties of 2D space crystals

5:1 resonances along x and y directions



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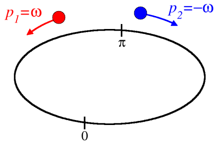
K. Giergiel, A. Miroszewski, KS, PRL **120**, 140401 (2018).

Time engineering

Anderson molecule

Two atoms bound together not due to attractive interaction but due to destructive interference

$$H = \frac{p_1^2 + p_2^2}{2} + \delta(\theta_1 - \theta_2)\, f(t) \quad \longrightarrow \quad H_{eff} = \frac{P_1^2 + P_2^2}{2} + \sum_k f_{-2k} e^{ik(\Theta_1 - \Theta_2)}$$



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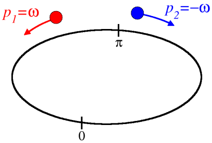
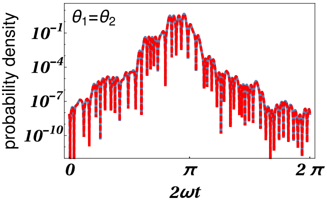




K. Giergiel, A. Miroszewski, KS, PRL **120**, 140401 (2018).

Summary:

1. Time crystals are analogues of space crystals but in the time domain.
2. Crystalline structures in time can emerge in dynamics of resonantly driven single- and many-particle systems.
3. **Periodically driven systems are platform for time crystal research:**
   - topological time crystals,
   - quasi-crystal structures in time,
   - many-body systems with exotic interactions,
   - time crystals with properties of 2D or 3D space crystals,
   - Anderson localization in the time domain induced by disorder in time,
   - many-body localization caused by temporal disorder,
   - dynamical quantum phase transition in time crystals.
4. Time engineering: Anderson molecule.

KS, PRA **91**, 033617 (2015).
KS, Sci. Rep. **5**, 10787 (2015).
KS, D. Delande, PRA **94**, 023633 (2016).
K. Giergiel, KS, PRA 95, 063402 (2017).
M. Mierzejewski, K. Giergiel, KS, PRB **96**, 140201 (2017).
D. Delande, L. Morales-Molina, KS, PRL **119**, 230404 (2017).
A. Syrwid, J. Zakrzewski, KS, PRL **119**, 250602 (2017).
K. Giergiel, A. Miroszewski, KS, PRL **120**, 140401 (2018).
A. Kosior, KS, PRA **97**, 053621 (2018).
K. Giergiel, A. Kosior, P. Hannaford, KS, PRA **98**, 013613 (2018).
A. Kosior, A. Syrwid, KS, arXiv:1806.05597.
K. Giergiel, A. Dauphin, M. Lewenstein, J. Zakrzewski, KS, arXiv:1806.10536.
K. Giergiel, A. Kuroś, KS, arXiv:1807.02105.

KS, J. Zakrzewski, **Time crystals: a review**, Rep. Prog. Phys. 81, 016401 (2018).

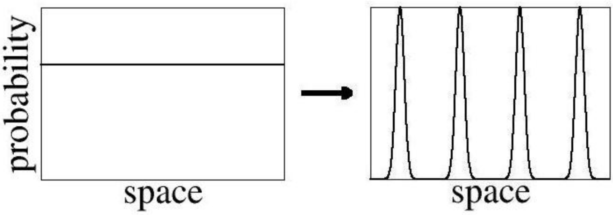
Formation of space crystals

$$[\hat{H}, \hat{T}] = 0$$

$\hat{H}$ – solid state system Hamiltonian
$\hat{T}$ – translation operator of all particles by the same vector

$$\left|\hat{T}\psi\right|^2 = \left|e^{i\alpha}\psi\right|^2 = |\psi|^2$$

t =const.

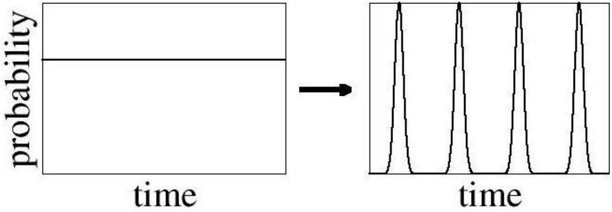


Formation of time crystals?

Eigenstates of a time-independent Hamiltonian H are also eigenstates of a time translation operator e^{-iHt}

$$\left|e^{-iHt}\psi\right|^2 = \left|e^{-iEt}\psi\right|^2 = |\psi|^2$$

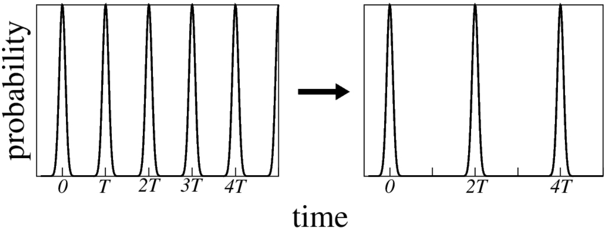
$\vec{r}$ is fixed



F. Wilczek, PRL **109**, 160401 (2012).
P. Bruno, PRL **111**, 070402 (2013).
H. Watanabe and M. Oshikawa, Phys. Rev. Lett. **114**, 251603 (2015).
A. Syrwid, J. Zakrzewski, KS, "*Time crystal behavior of excited eigenstates*", Phys. Rev. Lett. 119, 250602 (2017).

Discrete time crystals

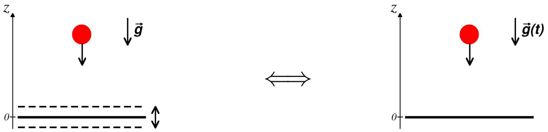
Spontaneous process



Discrete time crystals

Single particle bouncing on an oscillating mirror in 1D

Classically:



Floquet Hamiltonian:

$$H_F(t) = -\frac{1}{2}\frac{\partial^2}{\partial z^2} + z + \lambda z \cos(2\pi t/T) - i\frac{\partial}{\partial t}$$

$$H_F\ \psi_n(z,t) = E_n\ \psi_n(z,t)$$

E_n – quasi-energy

$\psi_n(z,t)$ – time periodic function

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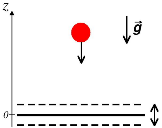
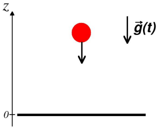


$\Longleftrightarrow$



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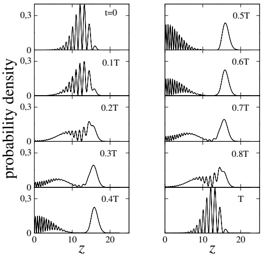
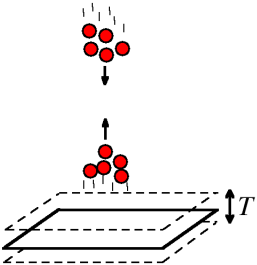
2 : 1 resonance

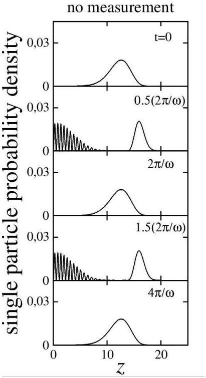


A. Buchleitner, D. Delande, J. Zakrzewski, Phys. Rep. **368**, 409 (2002).

Discrete time crystals

Bosons with attractive interactions





$N = 10^4$

$$|\psi\rangle \approx \frac{|N,0\rangle + |0,N\rangle}{\sqrt{2}}$$

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Discrete time crystals

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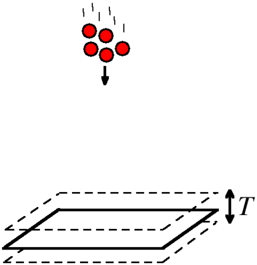

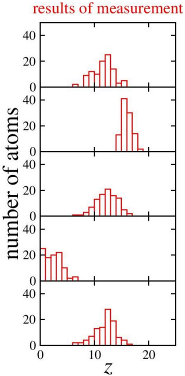

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$$|\psi\rangle \approx \frac{|N,0\rangle + |0,N\rangle}{\sqrt{2}} \longrightarrow |N-1,0\rangle \text{ or } |0,N-1\rangle$$

KS, Phys. Rev. A **91**, 033617 (2015).

Discrete time crystals

Spin systems

V. Khemani, A. Lazarides, R. Moessner, L. S. Sondhi, Phys. Rev. Lett. **116**, 250401 (2016).

D. V. Else, B. Bauer, C. Nayak, Phys. Rev. Lett. **117**, 090402 (2016).

LETTER

doi:10.1038/nature21413

Observation of a discrete time crystal

J. Zhang[1], P. W. Hess[1], A. Kyprianidis[1], P. Becker[1], A. Lee[1], J. Smith[1], G. Pagano[1], I.-D. Potirniche[2], A. C. Potter[3], A. Vishwanath[2,4], N. Y. Yao[2] & C. Monroe[1,5]

LETTER

doi:10.1038/nature21426

Observation of discrete time-crystalline order in a disordered dipolar many-body system

Soonwon Choi[1*], Joonhee Choi[1,2*], Renate Landig[1*], Georg Kucsko[1], Hengyun Zhou[1], Junichi Isoya[3], Fedor Jelezko[4], Shinobu Onoda[5], Hitoshi Sumiya[6], Vedika Khemani[1], Curt von Keyserlingk[7], Norman Y. Yao[8], Eugene Demler[1] & Mikhail D. Lukin[1]

Discrete time crystals

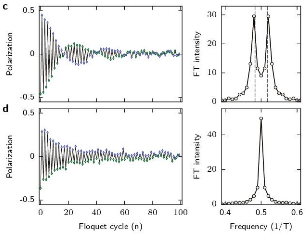
Spin systems

Chain of 10 ions: J. Zhang *et al.*, Nature (2017).

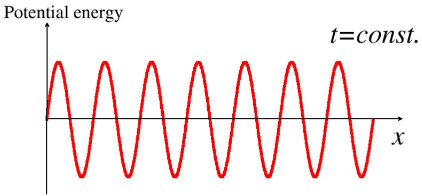
$$H = \begin{cases} H_1 = g(1-\varepsilon)\sum_i \sigma_i^y, & \text{time } t_1 \\ H_2 = \sum_i J_{ij}\sigma_i^x\sigma_j^x, & \text{time } t_2 \\ H_3 = \sum_i D_i\sigma_i^x & \text{time } t_3. \end{cases}$$

$$|\psi\rangle \approx \frac{|\uparrow\uparrow\ldots\uparrow\rangle_x \pm |\downarrow\downarrow\ldots\downarrow\rangle_x}{\sqrt{2}} \longrightarrow |\uparrow\uparrow\ldots\uparrow\rangle_x \quad \text{or} \quad |\downarrow\downarrow\ldots\downarrow\rangle_x$$

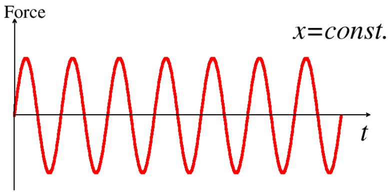
10^6 impurities in diamond: S. Choi *et al.*, Nature (2017).



Space crystals: $H(x + \lambda) = H(x)$



Time crystals: $H(t + T) = H(t)$



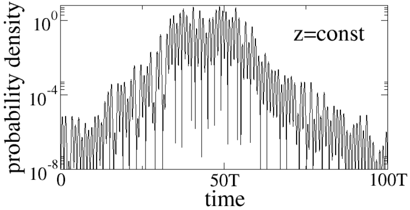
Anderson localization in the time domain

$$E_F = -\frac{J}{2}\sum_{j=1}^{s}(a_{j+1}^* a_j + \text{c.c.}) + \sum_{j=1}^{s}\varepsilon_j |a_j|^2$$

with $\varepsilon_j = \int_0^{sT} dt \langle \phi_j | H'(t) | \phi_j \rangle$,

where $H'(t)$ is a perturbation that fluctuates in time but $H'(t + sT) = H'(t)$.

Example for $s = 100$:



KS, Sci. Rep. **5**, 10787 (2015).

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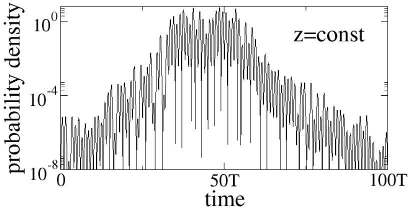
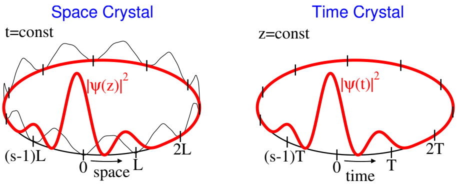




KS, Sci. Rep. **5**, 10787 (2015).

Mott insulator-like phase in the time domain

Ultra-cold atoms bouncing on an oscillating mirror

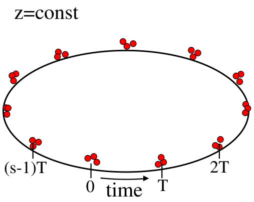
Bosons:

$$\hat{H}_F = -\frac{J}{2}\sum_{j=1}^{s}(\hat{a}_{j+1}^{\dagger}\hat{a}_j + \mathrm{h.c.}) + \frac{1}{2}\sum_{i,j=1}^{s} U_{ij}\ \hat{a}_i^{\dagger}\hat{a}_j^{\dagger}\ \hat{a}_j\hat{a}_i$$

with $U_{ij} = g_0\ \frac{2}{sT}\int_0^{sT} dt \int_0^{\infty} dz |\phi_i|^2\ |\phi_j|^2$.

- For $g_0 \to 0$, a superfluid state,
- For strong repulsion, $U_{ii} \gg NJ/s$, the ground state becomes a Fock state $|N/s, N/s, \ldots, N/s\rangle$.

Time Crystal



KS, Sci. Rep. **5**, 10787 (2015).

Many-body localization induced by temporal disorder

For example for bosons:

$$\hat{H}_F = -\frac{J}{2}\sum_{j=1}^{s}(\hat{a}_{j+1}^\dagger \hat{a}_j + \mathrm{h.c.}) + \sum_{j=1}^{s}\epsilon_j\ \hat{a}_j^\dagger \hat{a}_j + \frac{1}{2}\sum_{i,j=1}^{s} U_{ij}\ \hat{a}_i^\dagger \hat{a}_i\ \hat{a}_j^\dagger \hat{a}_j$$

with $U_{ij} \propto g_0 \int_0^{sT} dt \int_0^{\infty} dz\ |\phi_i|^2\ |\phi_j|^2$, where $|U_{ii}| > |U_{ij}|$ for $i \neq j$.

Many-body localization (MBL):

- vanishing of dc transport,
- absence of thermalization,
- logarithmic growth of the entanglement entropy,

It turns out that MBL can be also observed in the time domain due to the presence of temporal disorder.

M. Mierzejewski, K. Giergiel, KS, PRB **96**, 140201(R) (2017).

Recovery of Wilczek model

A. Syrwid, J. Zakrzewski, KS, arXiv:1702.05006

Bosons with attractive contact interactions on a ring:

$$H = \sum_{i=1}^{N} \frac{(p_i - \alpha)^2}{2} + \frac{g_0}{2} \sum_{i \neq j} \delta(x_i - x_j),$$

Mean field description: bright soliton solution.

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$$H = \frac{(P - N\alpha)^2}{2N} + \tilde{H}(\tilde{x}_i, \tilde{p}_i), \qquad P_j = 2\pi j$$

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$$\frac{\partial H}{\partial P_j} = 2\pi \frac{j}{N} - \alpha \approx 0$$

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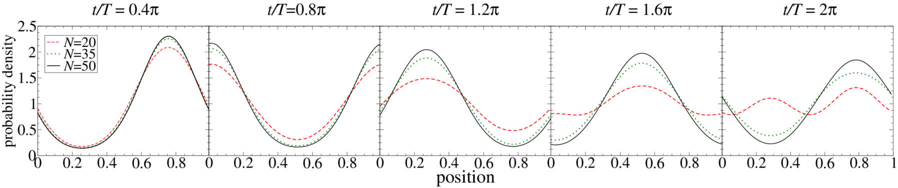
Excited state $P_N = 2\pi N$:

$$\frac{\partial H}{\partial P_N} = 2\pi - \alpha \neq 0$$

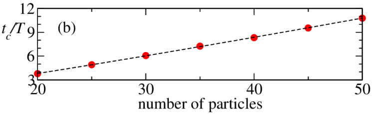
Recovery of Wilczek model

Measurement of the position x_1 of a single particle at $t = 0$ is expected to break continuous time translation symmetry:

$$\rho_2(x, t) \propto \langle \psi_0 | \hat{\psi}^\dagger(x, t)\hat{\psi}(x, t)\ \hat{\psi}^\dagger(x_1, 0)\hat{\psi}(x_1, 0)|\psi_0\rangle,$$



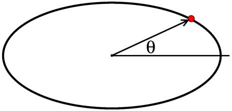
If the symmetry broken state lives forever in the limit $N \to \infty$, $g_0 \to 0$ with $g_0 N = \mathrm{const.}$, the time crystal is formed,



Anderson localization in the time domain

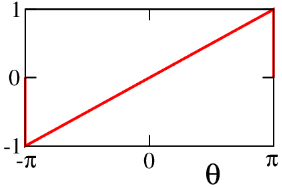
Single particle on a 1D ring

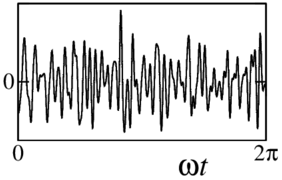
$$H = \frac{p^2}{2} + V\, g(\theta)\, f(t)$$



$$g(\theta) = \frac{\theta}{\pi} = \sum_n g_n e^{in\theta}$$

$$f(t + 2\pi/\omega) = f(t) = \sum_k f_k e^{ik\omega t}$$





KS, D. Delande, Phys. Rev. A **94**, 023633 (2016).

Anderson localization in the time domain

Single particle on a 1D ring

In the rotating frame $\tilde{\Theta} = \theta - \omega t$ is a **slow** variable if $\tilde{P} = p - \omega \approx 0$,

$$H_{eff} = \langle H(t) \rangle_t = \frac{\tilde{P}^2}{2} + V \sum_{k=-\infty}^{+\infty} g_k \; f_{-k} \; e^{ik\tilde{\Theta}}.$$

Eigenstates $\psi_n(\tilde{\Theta})$ of H_{eff} correspond to **Floquet states**, $(H(t) - i\partial_t)\psi_n = E_n \psi_n$,

where $\psi_n(\theta - \omega t)$ are time-periodic functions.

Anderson localization in the time domain

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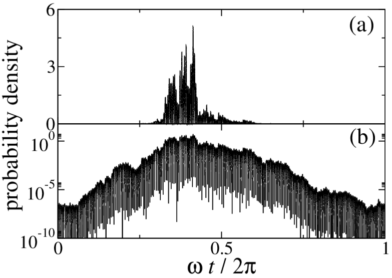
In the lab frame for fixed θ

For example $f(t)$ is so that $|g_k f_{-k}| \propto e^{-k^2/2k_0^2}$,
$Arg(f_k)$ is a random variable,
$k_0 = 10^3$, $V = 4 \cdot 10^3$, $E = 8 \cdot 10^3$.

Localization length in time $l_t = 0.17/\omega$.



KS, D. Delande, Phys. Rev. A **94**, 023633 (2016).

Phase transition in Anderson localization in time

Time crystals with properties of 3D systems

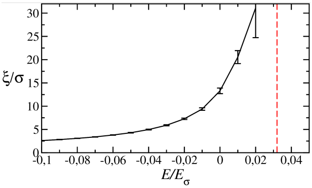
$$H = \frac{p_\theta^2 + p_\psi^2 + p_\phi^2}{2} + V_0 g(\theta) g(\psi) g(\phi) f_1(t) f_2(t) f_3(t),$$

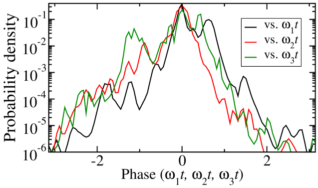
where $f_i(t + 2\pi/\omega_i) = f_i(t) = \sum_k f_k^{(i)} e^{ik\omega_i t}$.

In the moving frame, $\Theta = \theta - \omega_1 t, \ \Psi = \psi - \omega_2 t, \ \Phi = \phi - \omega_3 t,$

$$H_{\text{eff}} = \frac{P_\Theta^2 + P_\Psi^2 + P_\Phi^2}{2} + V_0 h_1(\Theta) h_2(\Psi) h_3(\Phi),$$

where $h_i(x) = \sum_k g_k f_{-k}^{(i)} e^{ikx}$ are disordered potentials.





D. Delande, L. Morales-Molina, KS, arXiv:1702.03591.

Discrete time crystals

Bosons with attractive interactions

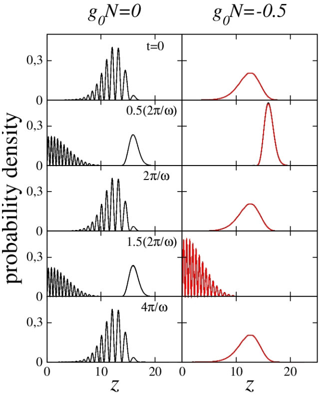
Gross-Pitaevskii equation:

$$\left[H_0 + g_0 N|\psi|^2 - i\partial_t\right]\psi = \mu\psi,$$

$H_0 = -\frac{1}{2}\partial_z^2 + z + \lambda z \cos(\omega t),$

where

$\psi(z,t)$ is a time periodic function.



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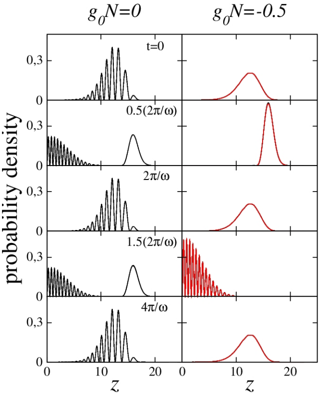


$$\psi \approx \phi_1(z,t)\, a_1 + \phi_2(z,t)\, a_2$$

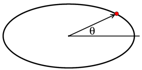
$$E = \int_0^\infty dz \int_0^{4\pi/\omega} dt\, \psi^* \left(H_0 - i\partial_t + \frac{g_0 N}{2}|\psi|^2\right)\psi \approx E(a_1, a_2),$$

KS, Phys. Rev. A **91**, 033617 (2015).

Condensed matter physics in the time domain

- Anderson localization in time without non-spreading wave-packets:

$$H = \frac{p^2}{2} + V\, g(\theta)\, f(t),$$



$g(\theta)$ is a regular function, $f(t)$ fluctuates randomly.

KS, D. Delande, Phys. Rev. A **94**, 023633 (2016).

- Anderson localization of an electron along a Kepler orbit in an Hydrogen atom perturbed by a fluctuating microwave field.

K. Giergiel, KS, Phys. Rev. A 95, 063402 (2017).

- Phase transition in Anderson localization in the time domain — time crystals with properties of 3D systems.

D. Delande, L. Morales-Molina, KS, arXiv:1702.03591.

- Many-body systems: Mott-insulator like phase and many-body localization in the time domain

KS, Sci. Rep. **5**, 10787 (2015).
M. Mierzejewski, K. Giergiel, KS, arXiv:1706.09791.

Maximal number of states localized in a s-resonant island:

$$n_{max} \approx s\frac{8\sqrt{\lambda}}{\omega^3}.$$

Resonant action

$$I_s = s^3\frac{\pi^2}{3\omega^3}.$$